Overset Grid Symposium 2010

Spectral FC Solvers on Overset Grids

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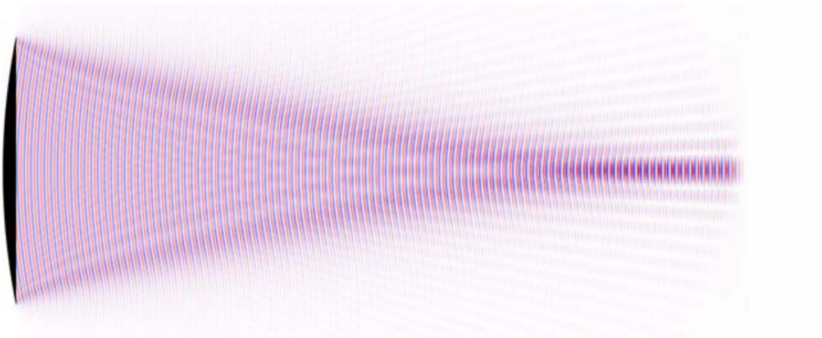
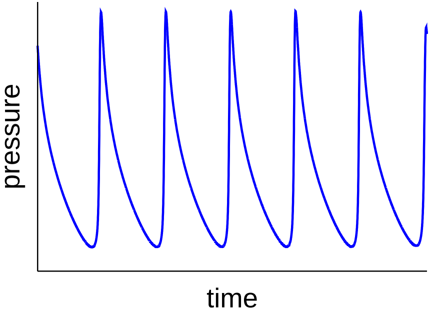
joint work with Oscar P. Bruno

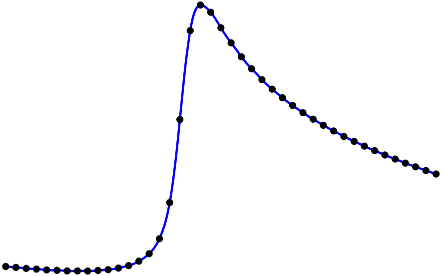
California Institute of Technology

How do you solve hundred-wavelength acoustic problems?

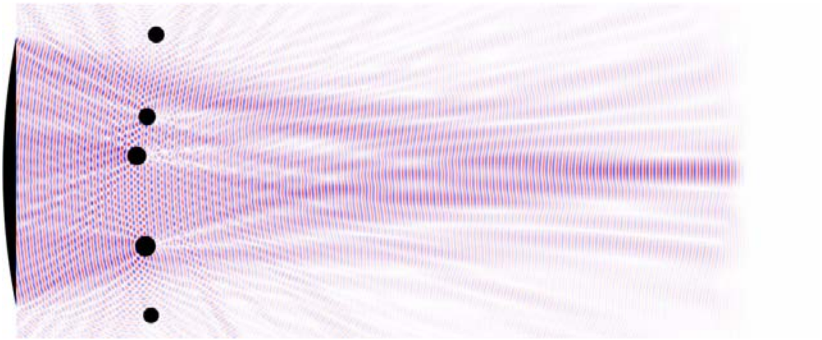


...with nonlinear effects?

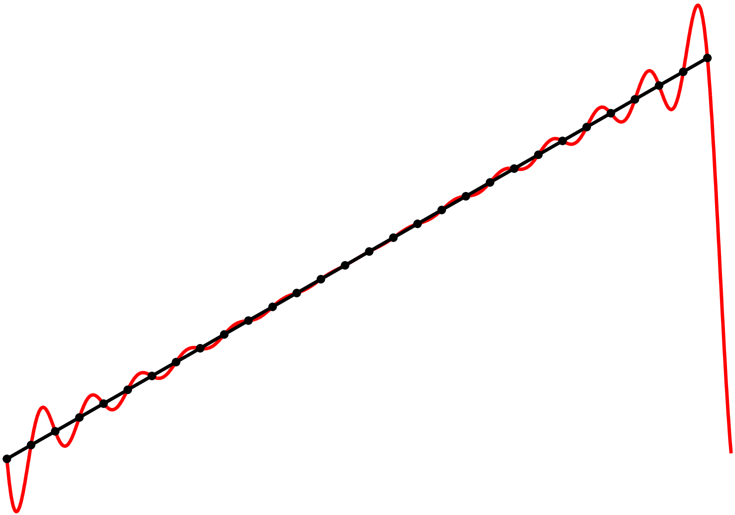




...in complex geometries?

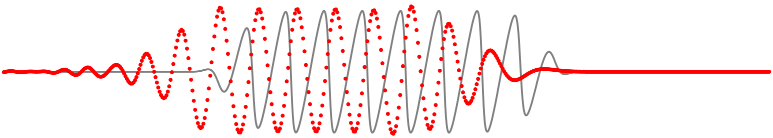


...with a Fourier spectral method?

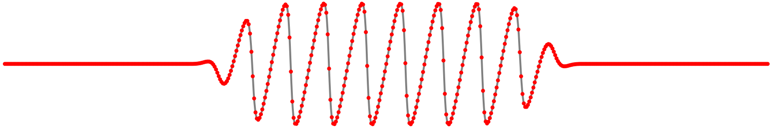


Why?

Fourier spectral methods have small dispersion errors



2^{nd}-order FD



Fourier Spectral

Fourier spectral methods have mild CFL conditions

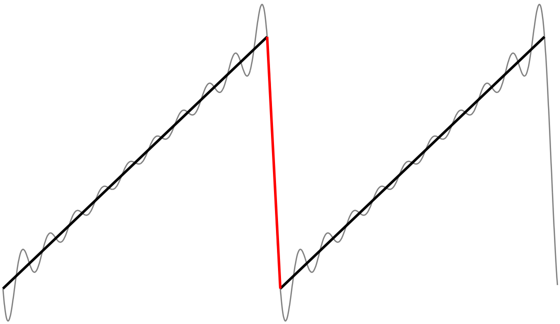
$$\Delta t \sim \Delta x^{2}$$

instead of

$$\Delta t \sim \Delta x^{4}$$

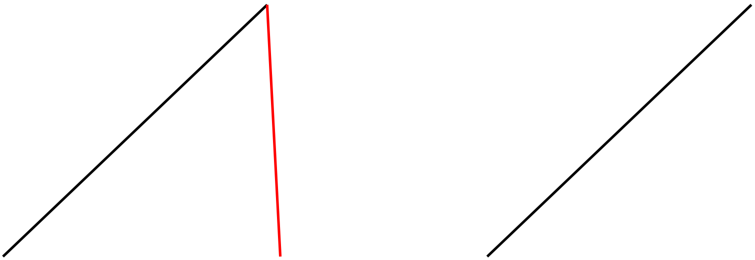
How?

Problem: Fourier series has poor convergence at jumps



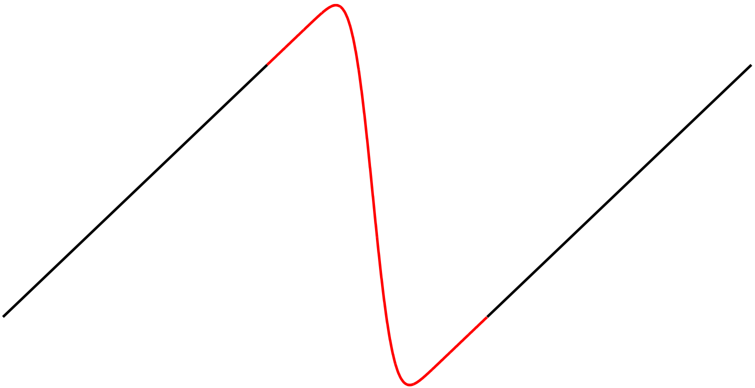
$$f(x) \approx \sum_{k=-N}^{N} \hat{f}_k \exp\left(\frac{2\pi i k x}{1}\right)$$

Solution: Fourier Continuation (FC) eliminates jumps



$$f(x) \approx \sum_{k=-N}^{N} \hat{f}_k \exp\left(\frac{2\pi i k x}{b}\right)$$

Solution: Fourier Continuation (FC) eliminates jumps



$$f(x) \approx \sum_{k=-N}^{N} \hat{f}_k \exp\left(\frac{2\pi i k x}{b}\right)$$

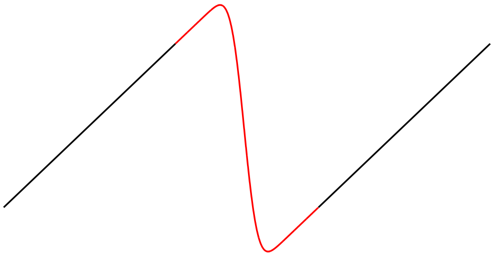
Properties

FC is accurate

# grid points	max. err. in $f'(x)$
20	2.46(−1)
40	5.72(−4)
80	4.08(−9)
160	6.11(−14)

$$f(x) = \frac{1}{1 + 25x^2} \qquad x \in [-1, 1]$$

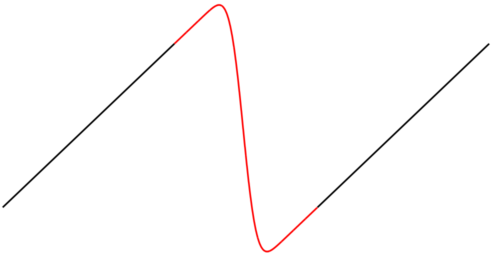
FC is fast



1D Derivative Algorithm

1. FC $O(1)$
2. FFT $O(N \log N)$
3. Derivative $O(N)$
4. iFFT $O(N \log N)$
5. Restrict $O(1)$

FC is fast



1D Derivative Algorithm

1. FC $O(1)$
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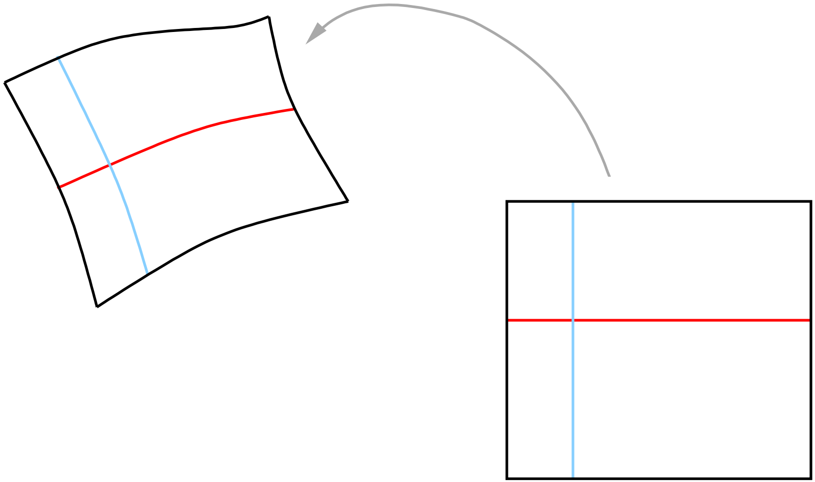
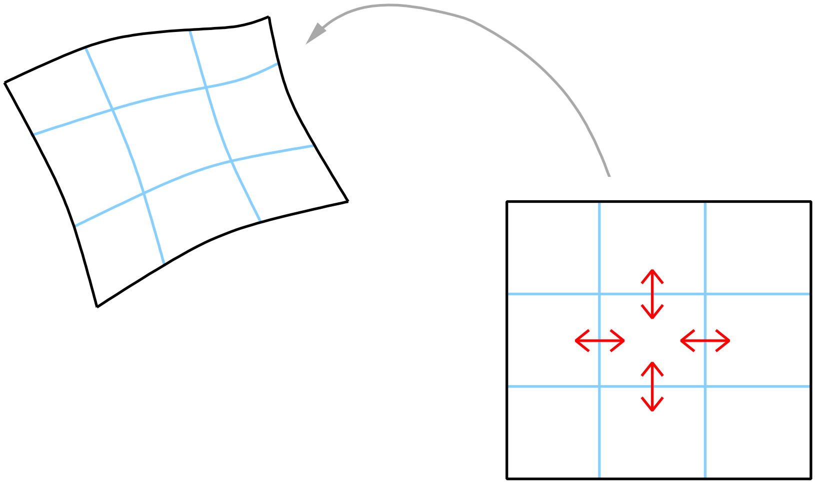
5. Restrict $O(1)$

$\sim 5\%$
of CPU time

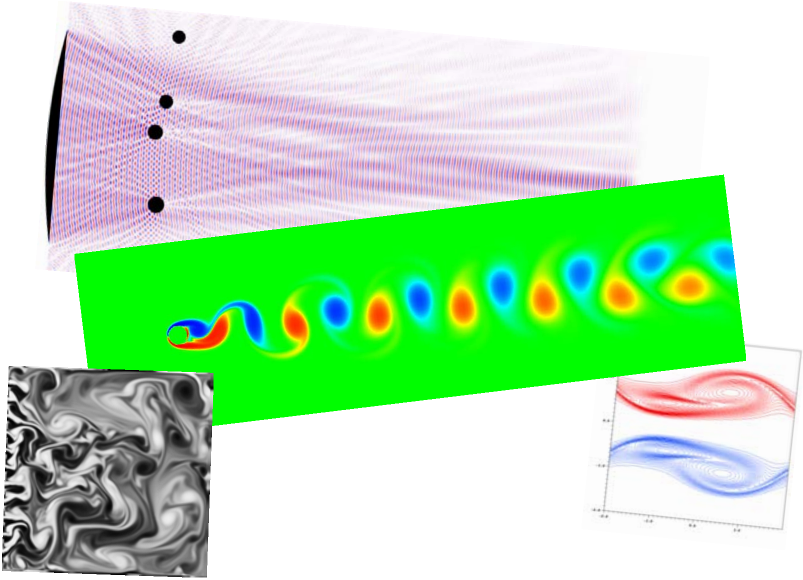
FC works in multiple dimensions (line-by-line)



FC gives efficient parallelization



FC gives high-order solvers for nonlinear PDEs



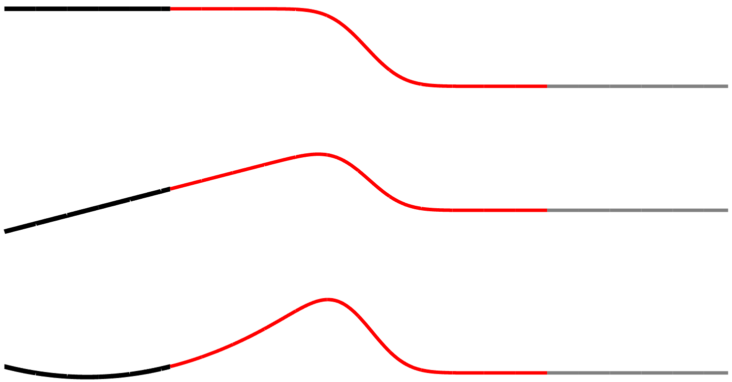
Details

FC is a fast, $O(1)$ operation
(computed from near-boundary values)

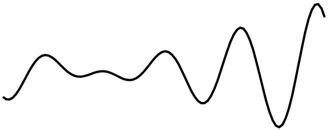
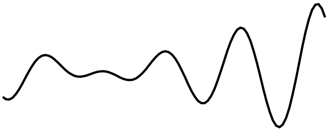
$$\begin{bmatrix} f \\ f_c \end{bmatrix} = \left[\begin{array}{ccc} & \mathrm{I} & \\ \hline C_\ell & 0 & C_r \end{array}\right] \times f$$

C_ℓ and C_r are fixed matrices

FC is based on smooth extensions of a polynomial basis

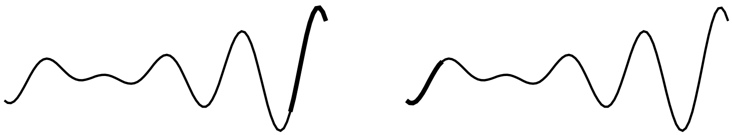


FC by numbers

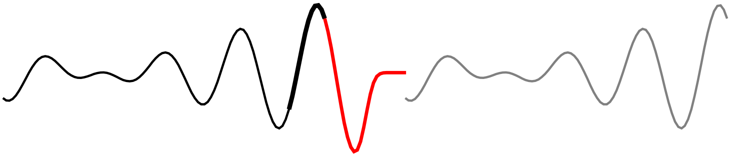
FC by numbers

Step 1: Boundary interpolation



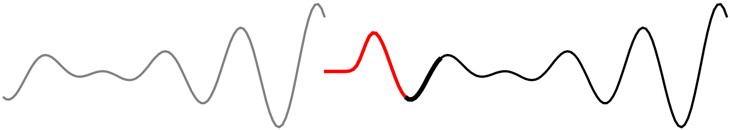
FC by numbers

Step 2a: Right continuation



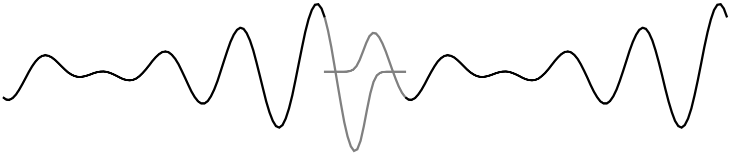
FC by numbers

Step 2b: Left continuation



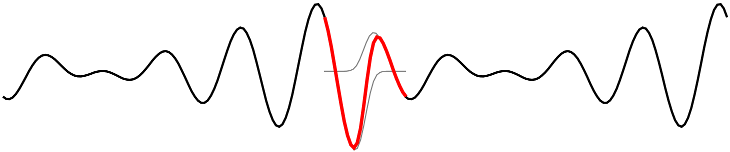
FC by numbers

Step 3: Add



FC by numbers

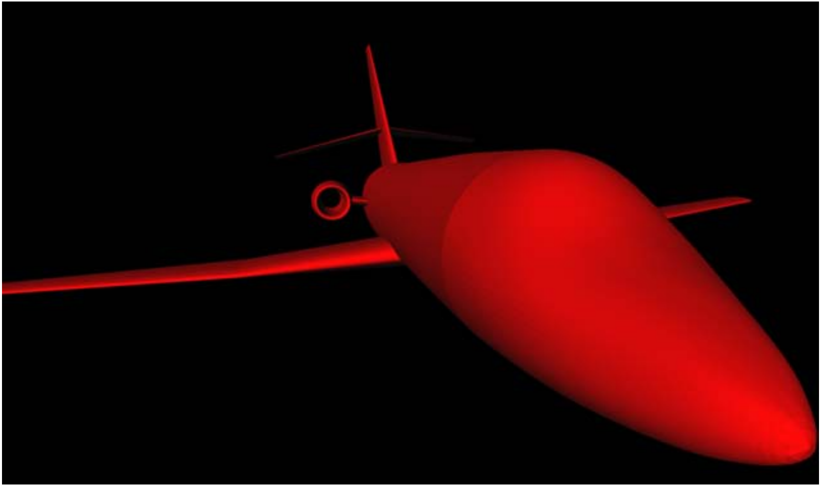
Step 3: Add



Other uses of FC

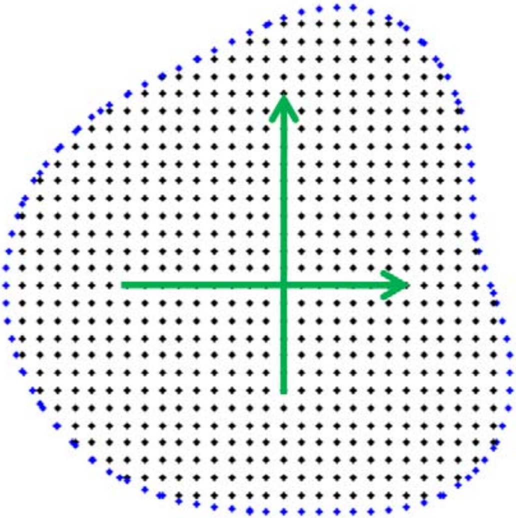
High-order surface representation

O. Bruno, Y. Han and M. Pohlman



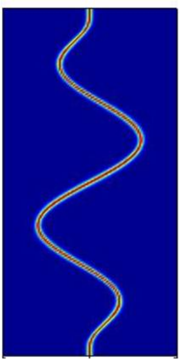
ADI-type solvers

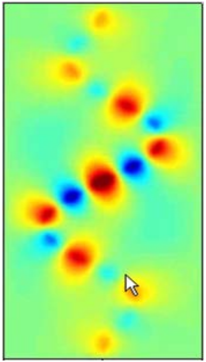
O. Bruno and M. Lyon



ADI for variable coefficient problems

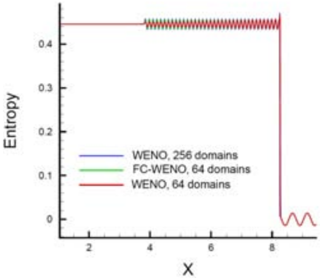
O. Bruno and A. Prieto

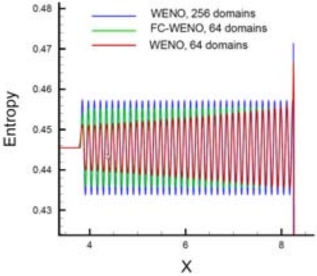




FC/WENO hybrid

N.A., O. Bruno, J. Hesthaven and K. Shahbazi





The moral of the story: FC solvers are...

- Fast
  $O(N \log N)$ complexity of FFT
- Accurate
  high-order + spectral away from boundaries
- Efficiently parallel
  linear scaling
- Versatile
  successfully applied to a variety of 2D and 3D problems
- Easy to implement
  linear continuation operator + standard methods