10th Symposimum on Overset Composite Grids and Solution Technology, NASA Ames Research Center Moffett Field, California, USA

Implementing a Partitioned Algorithm for Fluid-Structure Interaction of Flexible Flapping Wings within *Overture*

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Work Done at

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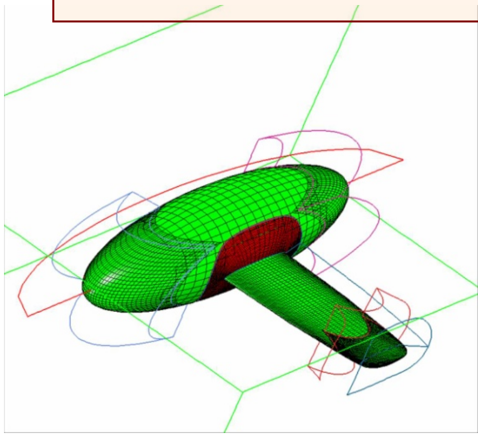
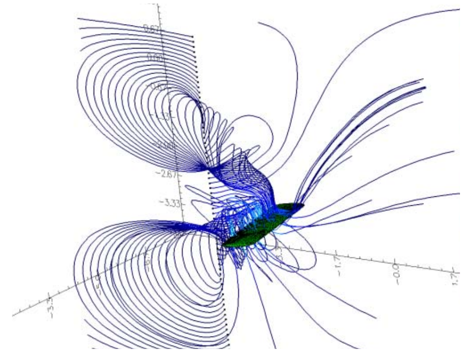
Outline

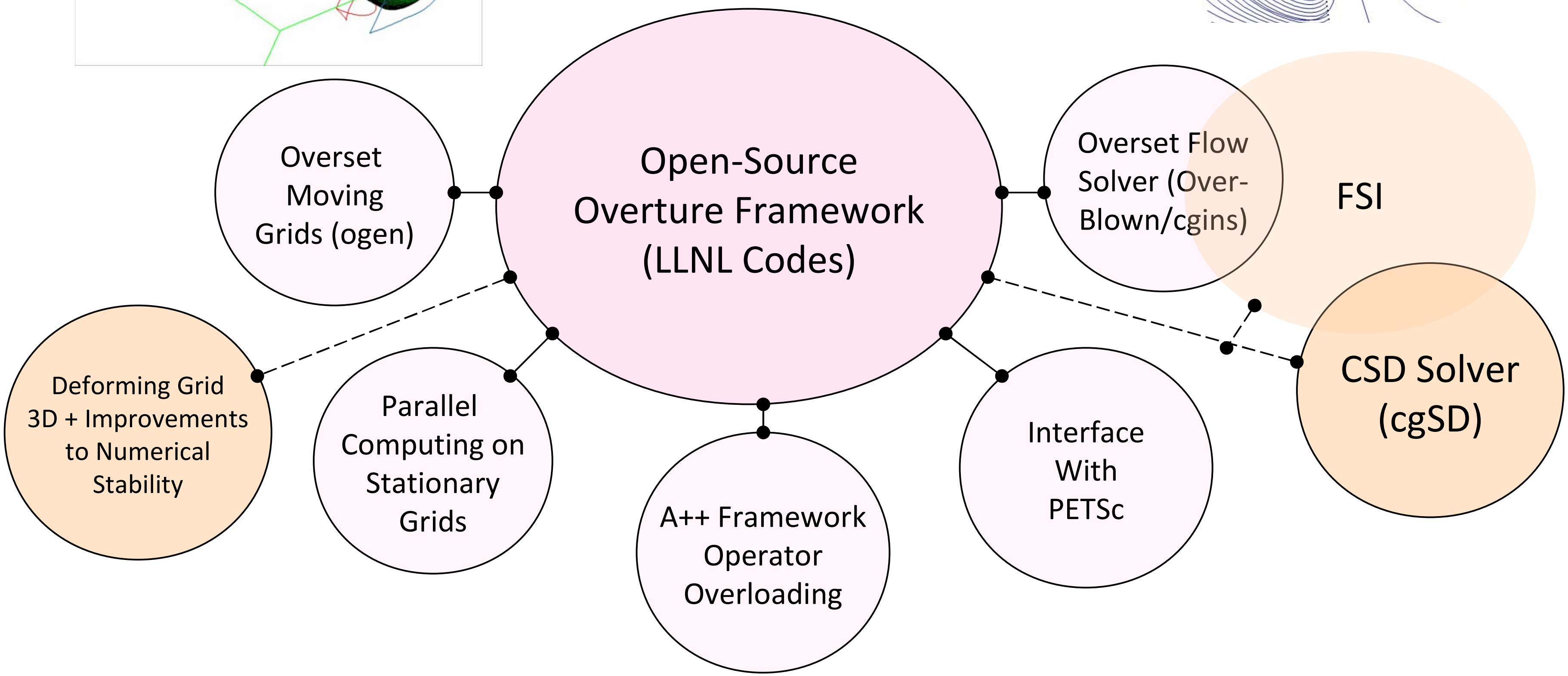
- → Overture Framework
- → Definition of various problems
- → Computational Framework
  - ➢ Overlapping grids
  - ➢ Fluid dynamics
  - ➢ Structural dynamics
  - ➢ Rigid body dynamics
- → Key results and discussions
- → Concluding remarks

Overture Framework

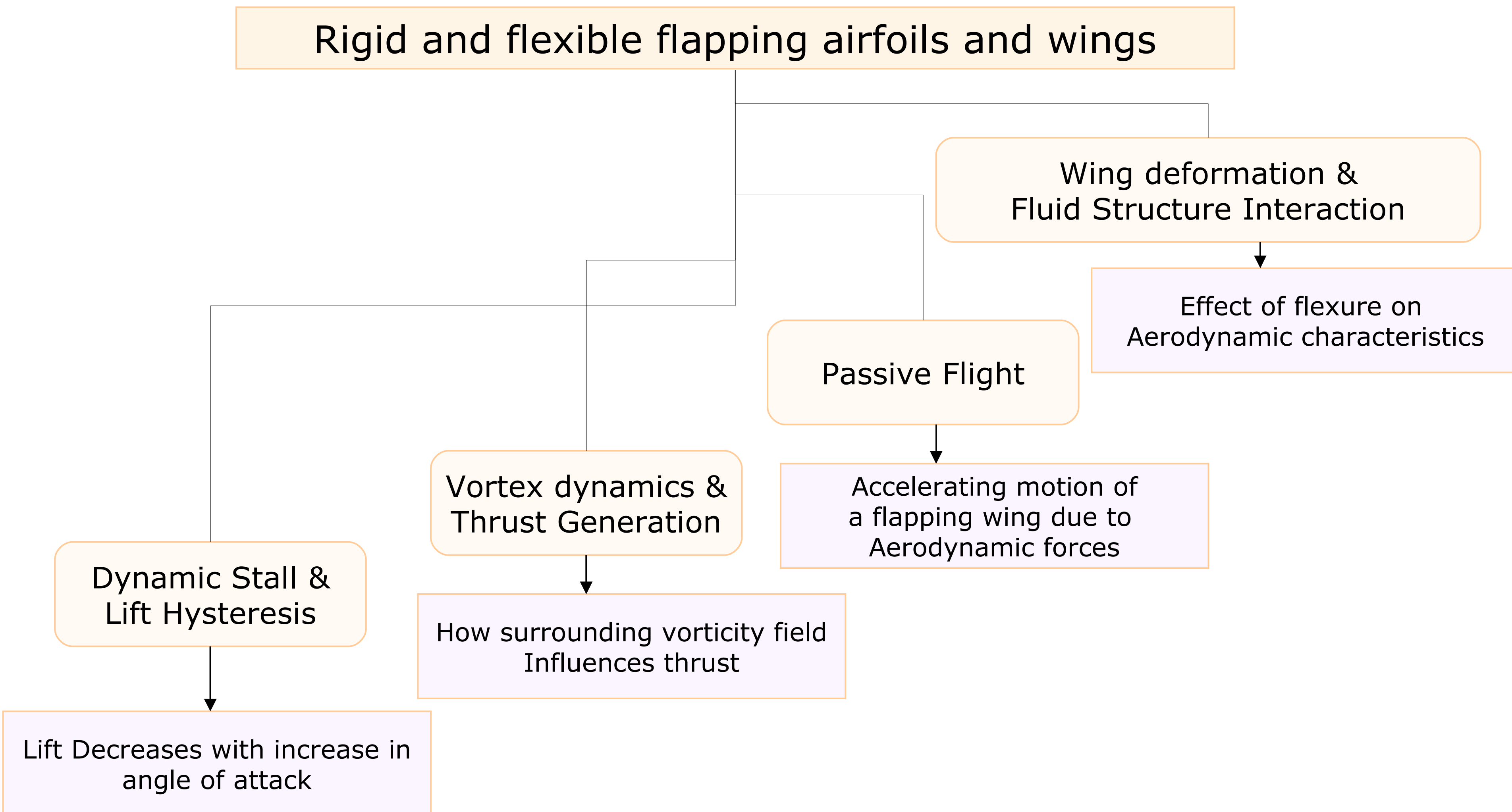


Overture / OverBlown (CGINS) Codes



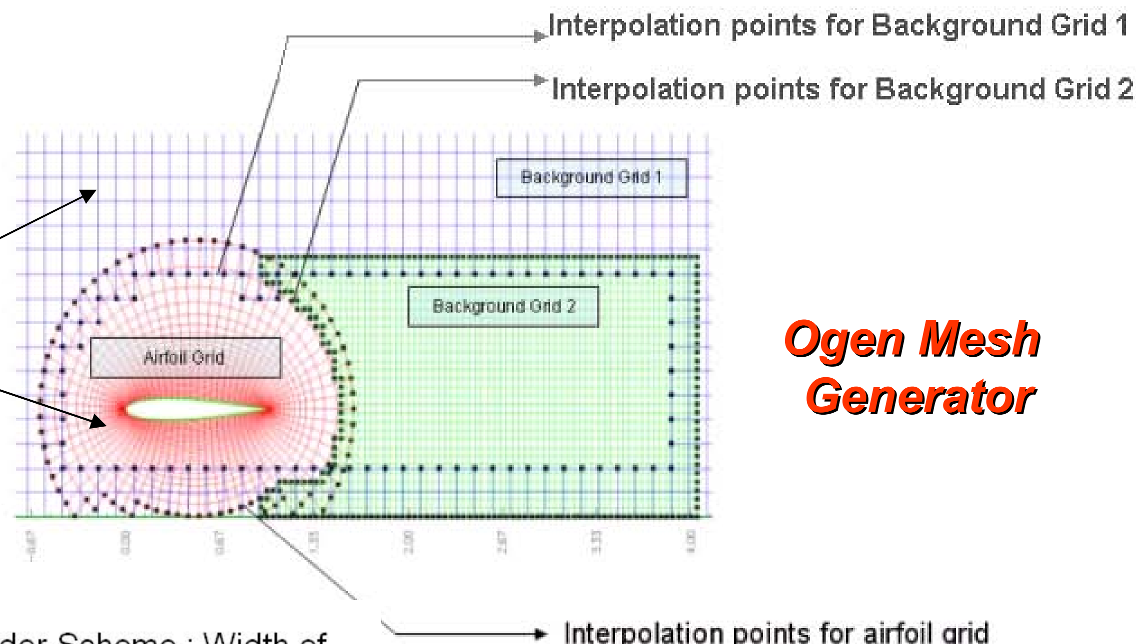


Summary of Work Undertaken using Overture



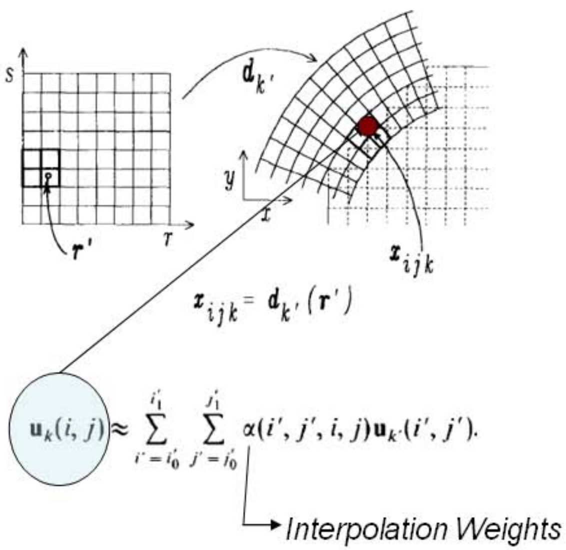
Overlapping / Composite / Overset / Chimera Grids

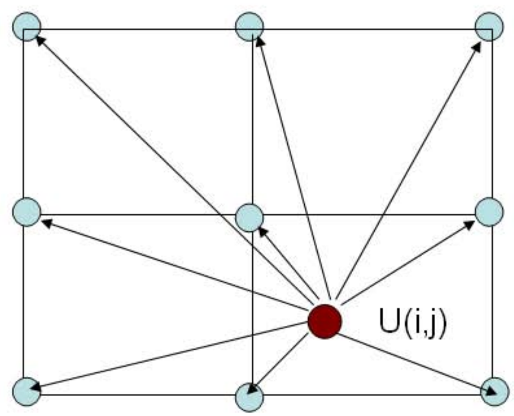
Individual grids can be Generated independently



Ogen Mesh Generator

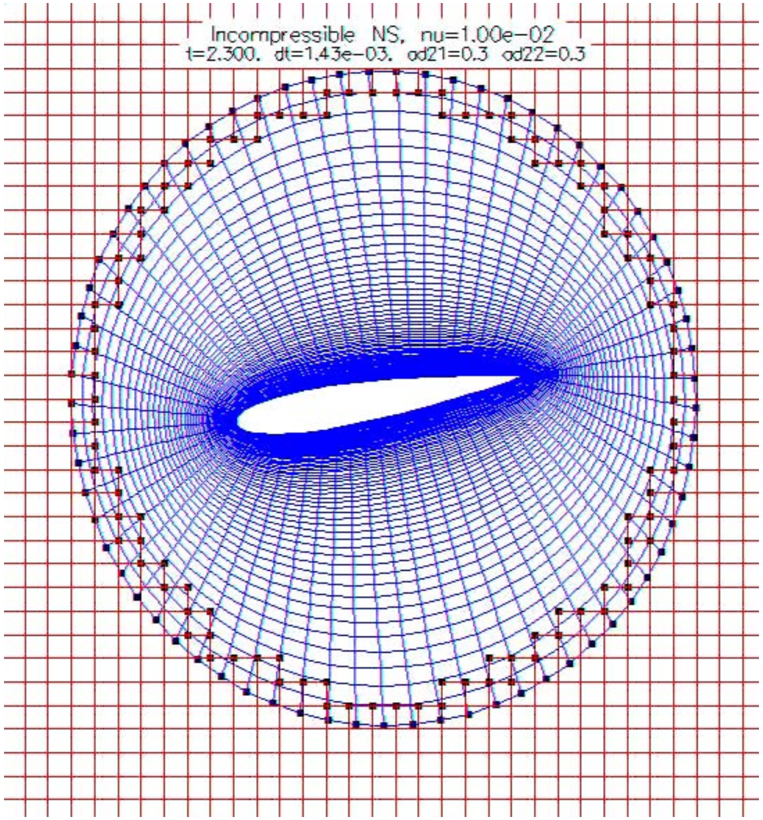
For 2[nd] order Scheme : Width of Interpolation = 3 (Quadratic)



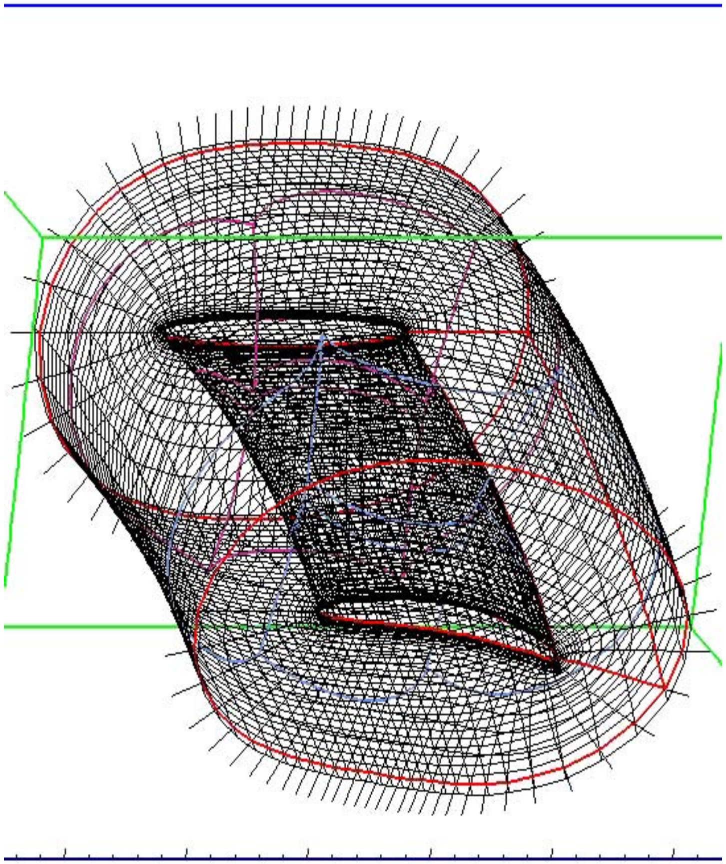


Interpolation

Moving Overlapping Grids



Two-Dimensional Moving Grid (Rigid)



Three-Dimensional Deforming Grid

Computational Flow Modeling

(A) Fluid Dynamics – Incompressible Navier Stokes Equations

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\frac{\nabla p}{\rho} + \nu \Delta \mathbf{u}$$

$$\nabla \cdot \mathbf{u} = 0$$

u → Velocity Vector

p → Static pressure

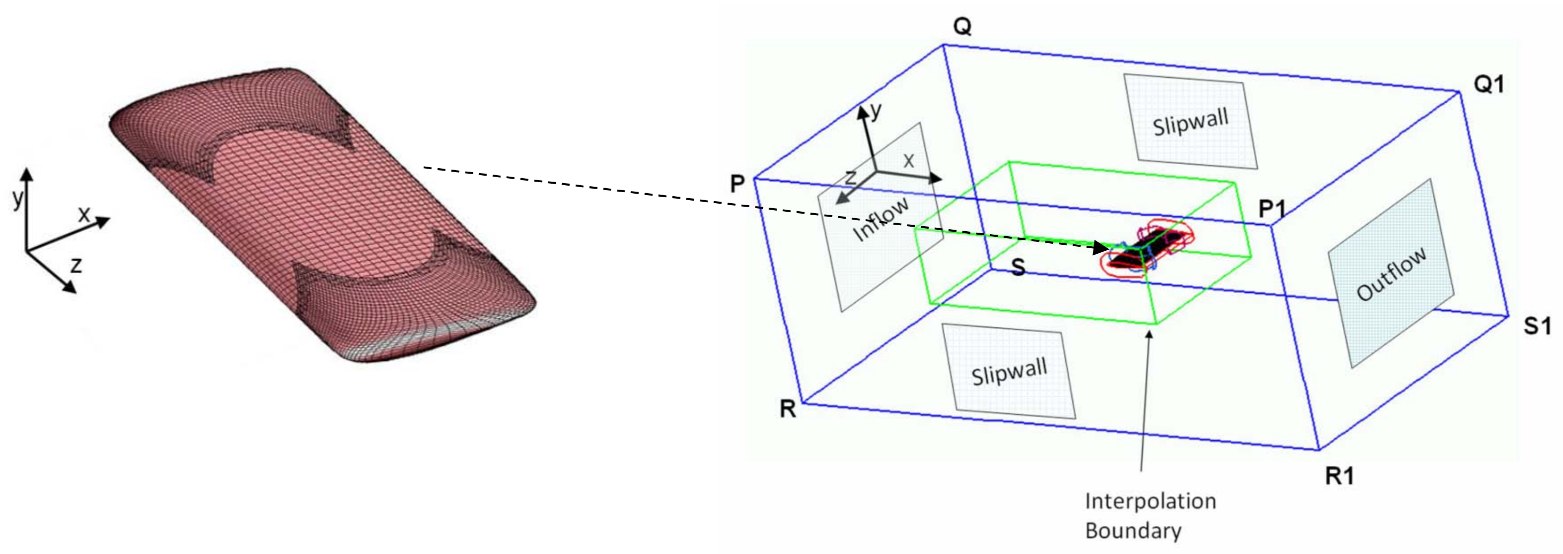
ρ → Fluid density

Pressure Poisson Equation

$$\Delta p - (\nabla u \cdot \mathbf{u_x} + \nabla v \cdot \mathbf{u_y} + \nabla w \cdot \mathbf{u_z}) = 0$$

- ✓ 2nd Order spatial differences
- ✓ 2nd Order Crank Nicolson Implicit (For Viscous terms)
- ✓ 2nd Order Adams Predictor-Corrector (Explicit)

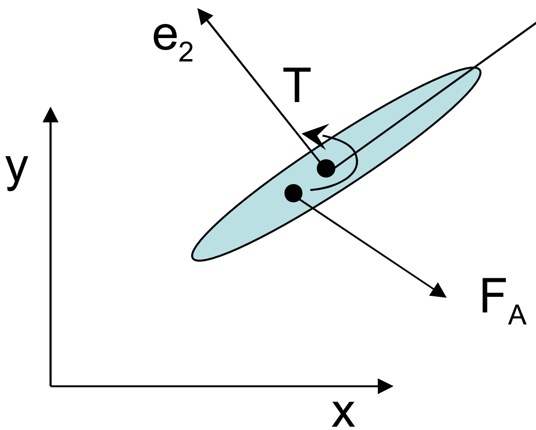
Computational Flow Modeling



Boundary condition type	Region	Boundary condition
Wall (No slip)	Wing surface	$u = 0,\ \nabla . u = 0, \dfrac{\partial p}{\partial n} = n \cdot \left(-\ddot{G}_t - \upsilon \nabla \times \nabla \times \mathbf{u}\right)$
Far field	P-R-R1-P1-P Q-S-S1-Q1-Q Q-P-P1-Q1-Q S-R-R1-S1-S	$n.u = 0,\ \dfrac{\partial}{\partial n}\left(t_m . u\right) = 0,\ \nabla . u = 0$ (Slip wall conditions)
Inflow	P-Q-S-R-P	$u = u_s$ (*velocity specified*), $\dfrac{\partial p}{\partial n} = 0$
Outflow	P1-Q1-S1-R1-P1	*Extrapolate* u, $\dfrac{\partial p}{\partial n} = 0$

Computational Modeling

(B) 6 DOF Rigid Body Dynamics



F_A= Net Aerodynamic Force

T = Net Aerodynamic Torque / Moment about Centre of mass (x_{cms})

M = Mass of the body

I = Components of Principal moments of Inertia

ω= Angular velocity vector

e_i = Principal axes.

τ = Stress Tensor

$$F_{A,i} = \int_{d\Omega} pn_i - \tau_{ki} n_k dS$$

Position

$$M \frac{d^2 \mathbf{x}_{cm}}{dt^2} = \mathbf{F}_A$$

$$\mathbf{I} \frac{d\omega}{dt} = \mathbf{T}$$

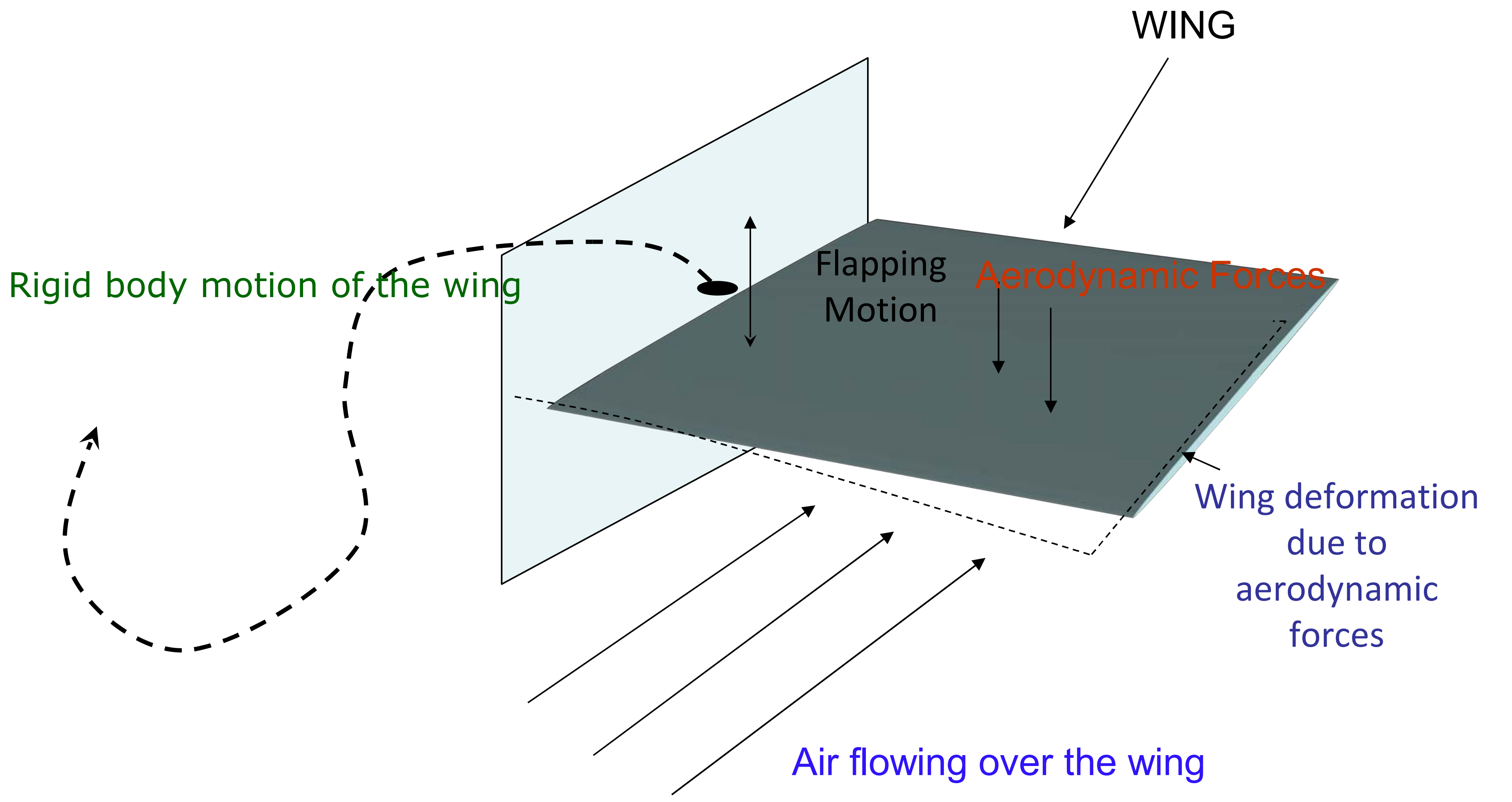
$$T = \int_{d\Omega} (r - x_{cm}) \times dF$$

$$\dot{e}_i = \omega \times e_i$$

$$I_i \dot{\omega}_i = \mathbf{T} \cdot e_i$$

Orientation

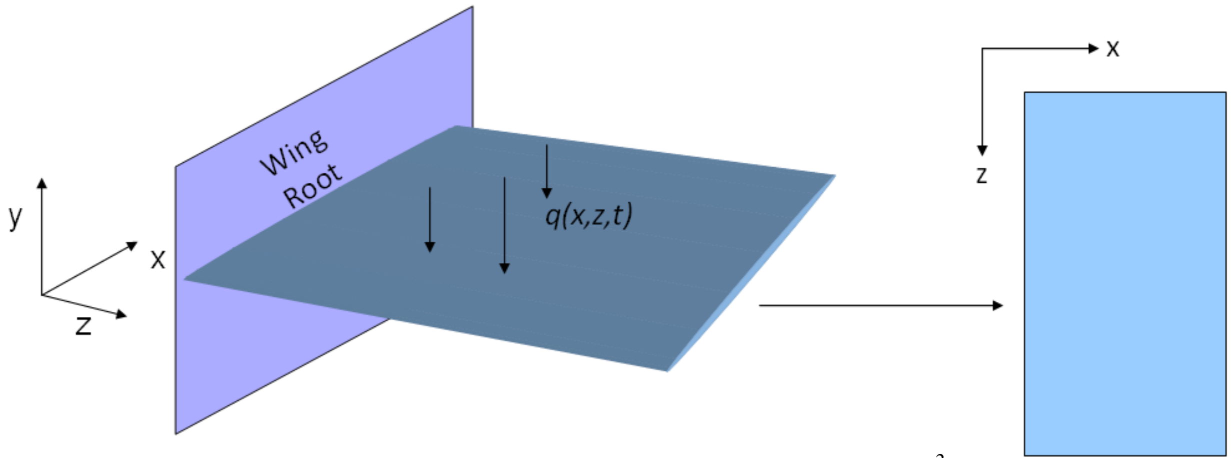
Computational Fluid-Structure Interaction and Coupling Issues



Computational Structural Dynamics Modeling

Structural Dynamics

Kirchhoff's Plate Equation



3D

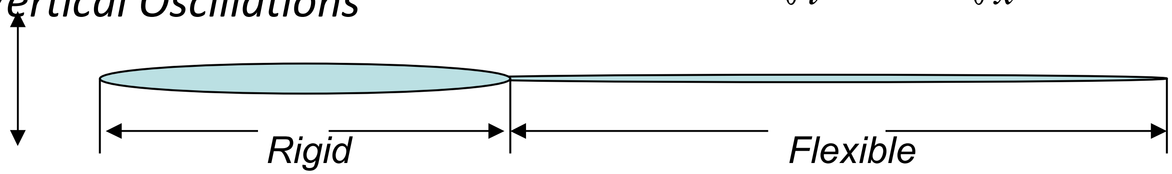
$$m_{xz} w_{tt} + D\nabla^4 w = q(x,z,t) \quad D = \frac{Eh^3}{12\left(1-\nu^2\right)}$$

Euler-Bernoulli Beam Equation

$$m_x \frac{\partial^2 w}{\partial t^2} + EI \frac{\partial^4 w}{\partial x^4} = q(x,t)$$

2D

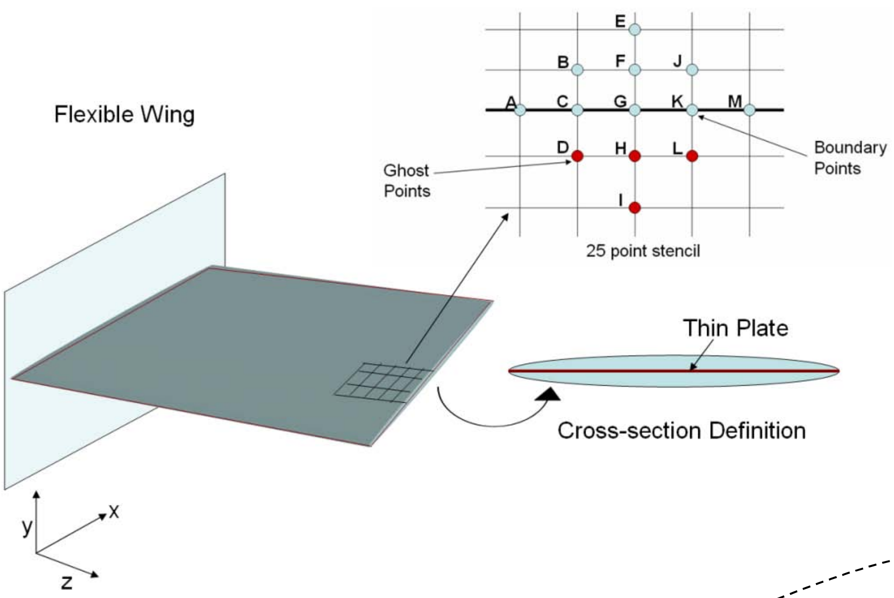
Vertical Oscillations



E : Modulus of Elasticity , *h* : Plate Thickness, ν : Poissons Ratio, m_{xz} : Mass per unit area, *q*: Load Acting, I: Moment of Inertia

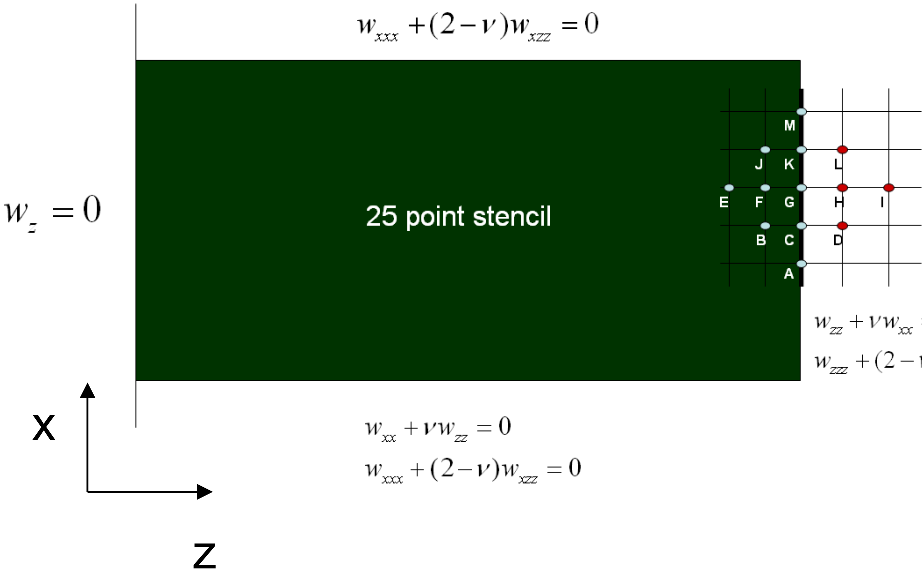
Computational Structural Dynamics Modeling

Structural Dynamics



$$w_{xx} + \nu w_{zz} = 0$$
$$w_{xxx} + (2-\nu) w_{xzz} = 0$$

$$w_z = 0$$



$$w_{zz} + \nu w_{xx} = 0$$
$$w_{zzz} + (2-\nu) w_{zxx} = 0$$

$$w_{xx} + \nu w_{zz} = 0$$
$$w_{xxx} + (2-\nu) w_{xzz} = 0$$

$$\kappa \dot{w}$$

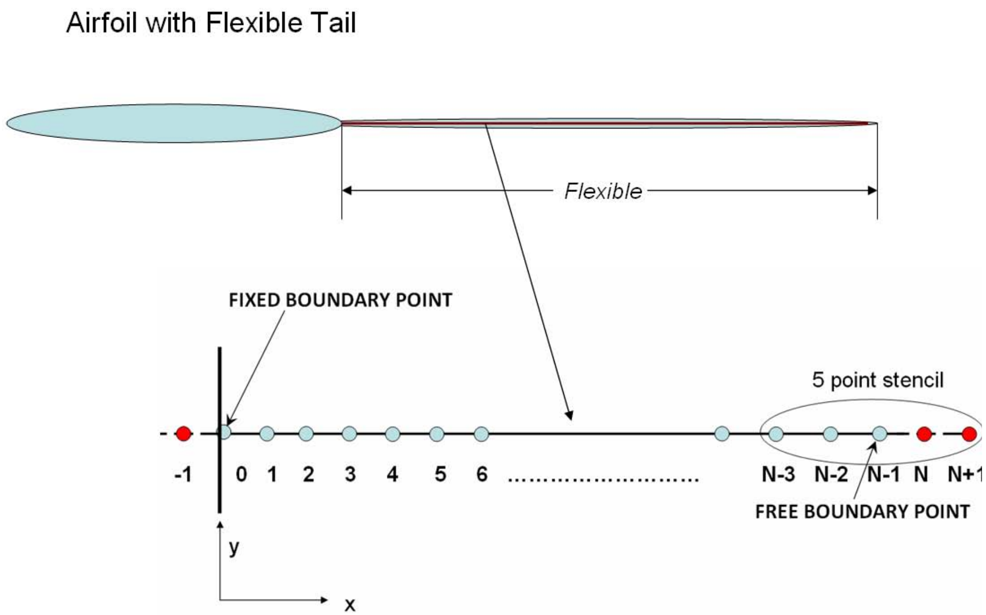
Governing Eqn → $$m_{xz} w_{tt} + D\nabla^4 w = q(x,z,t)$$

Static Part → $$\nabla^4 w = \sum_{i=1}^{13} A_i w_i = [Q]w, \quad A_i = A, B \ldots M$$

Dynamic Part → $$\left(I + \frac{\Delta t^2 D}{m_{xz}} [Q] \right) w^{n+1} = \frac{\Delta t^2 q(x,z,t)^{n+1}}{m_{xz}} + 2w^n - w^{n-1}$$

Computational Structural Dynamics Modeling

Discretization of the Euler-Bernoulli Beam Equation :



$$m_x \frac{w_i^{n+1} - 2w_i^n + w_i^{n-1}}{\Delta t^2} + EI \frac{w_{i+2}^{n+1} - 4w_{i+1}^{n+1} + 6w_i^{n+1} - 4w_{i-1}^{n+1} + w_{i-2}^{n+1}}{\Delta x^4} = q(x,t)^{n+1}$$

Solution → $[A]y^{n+1} = f(y^n, q^{n+1}) \qquad O(\Delta t, \Delta x^2)$

Fluid – Structure Coupling

Partitioned Approach *(Dirichlet – Neumann Approach)* With Inner Iterations :

Time = T^n

$$\frac{u^p - u^n}{\Delta t} = f(u^{n-1}, u^n, u^p, p^{n-1}, p^n)$$

$$\Delta p^p = f(u^p)$$

Transfer stresses through interpolation

$$[A]w^p = f(w^n, q^{n+1})$$

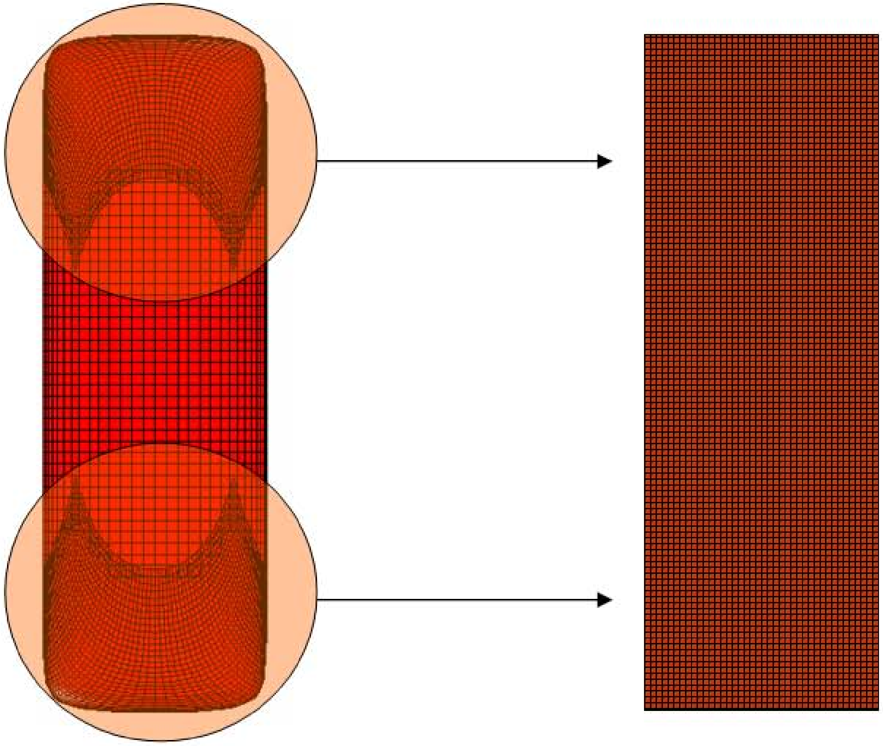
Time = T^{n+1}

Fluid – Structure Coupling

Load Transfer from Fluid to Solid:

Interpolate Wing Caps on to Wing

STEP 1



Fluid – Structure Coupling

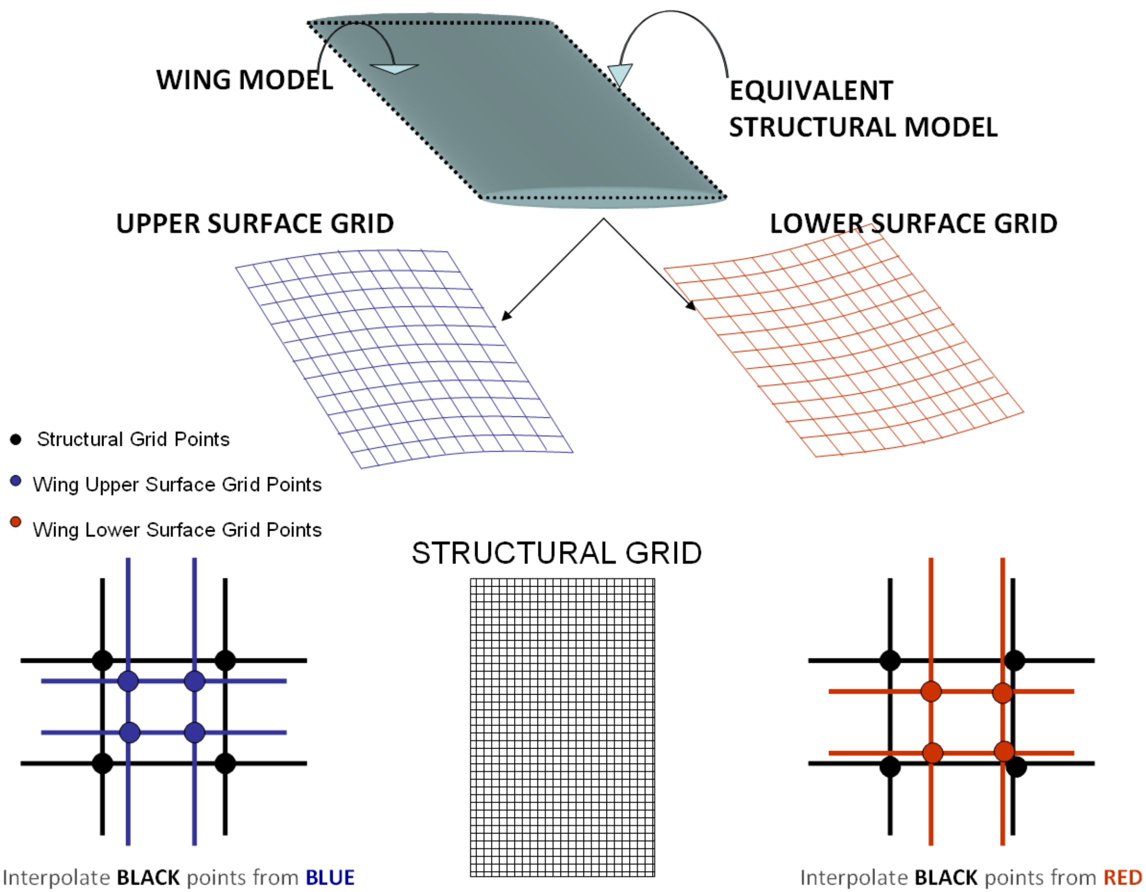
Stress Transfer from Fluid to Solid:

(Upper) (Lower)

$$q(x,z,t) = \Pi_u n_u + \Pi_l n_l$$

$$\Pi = p\delta_{ij} - \tau_{ij}$$

Interpolate CSD Grid from CFD

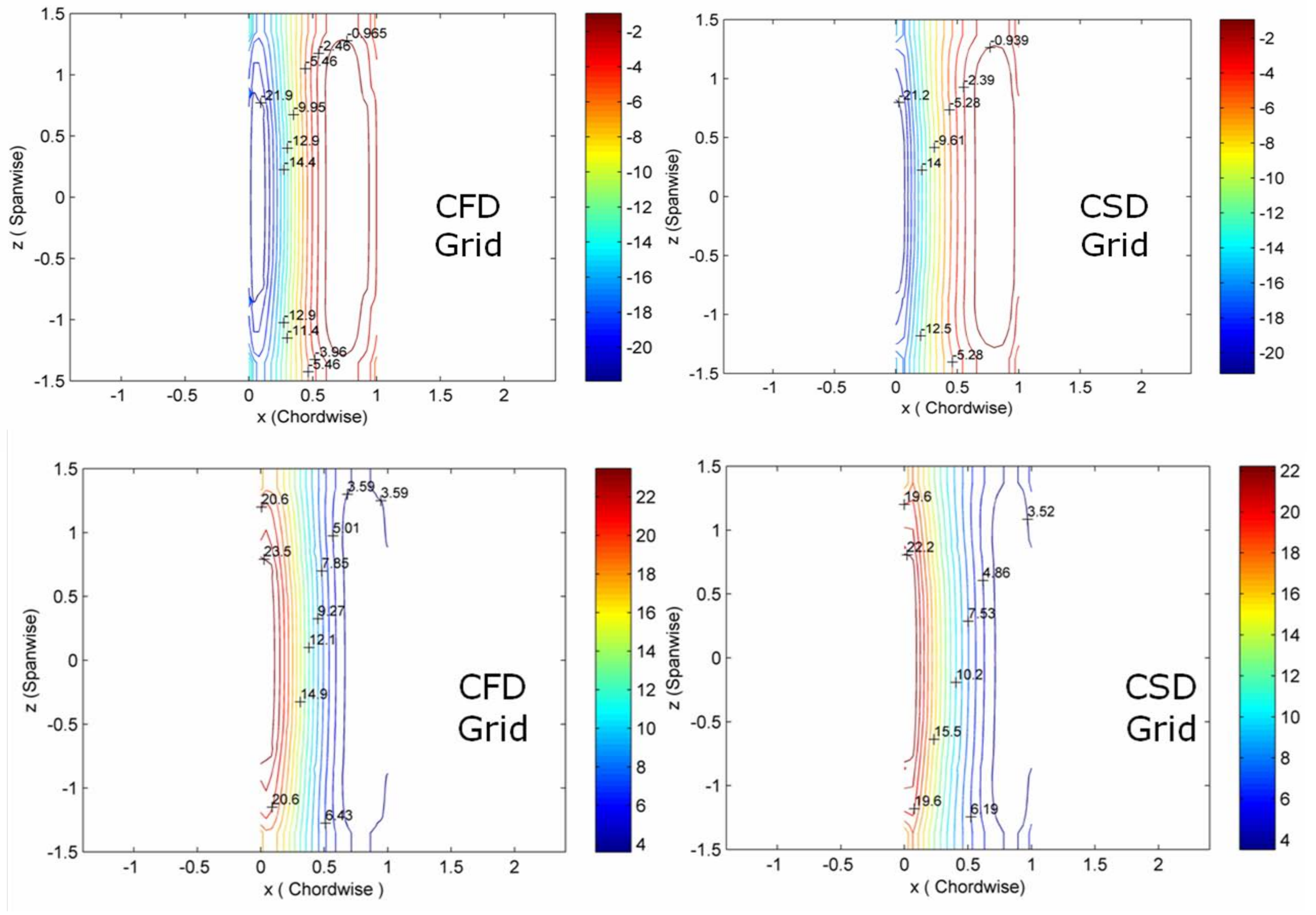


Fluid – Structure Coupling
Comparison of Actual and Interpolated Stress

Actual (CFD) | Interpolated (CSD)

Upper Surface

Lower Surface



Fluid – Structure Coupling

Partitioned Approach *(Dirichlet – Neumann Approach)* With Inner Iterations :

Time = T^n

$$\frac{u^p - u^n}{\Delta t} = f(u^{n-1}, u^n, u^p, p^{n-1}, p^n)$$

$$\Delta p^p = f(u^p)$$

Transfer stresses through interpolation

$$[A]w^p = f(w^n, q^{n+1})$$

Transfer displacements through interpolation

Local Iterations

$$\frac{u^c - u^n}{\Delta t} = f(u^p, u^n, u^c, p^p, p^n)$$

$$\Delta p^c = f(u^c)$$

$$[A]w^c_{k+1} = f(w^n, q^c)$$

How to choose ω

$$w^c_{k+1} = \omega w^c_{k+1} + (1-\omega) w^c_k$$

$$|w^c_{k+1} - w^c_k| < 10^{-9}$$

$$w^{n+1} = w^c_{k+1}, u^{n+1} = u^c, p^{n+1} = p^c$$

Time = T^{n+1}

Computational Cases Investigated

Rigid Plunging Wing

Plunging and (active) deforming airfoil

Plunging and (passive) deforming airfoil

Deformation of a beam in a fluid

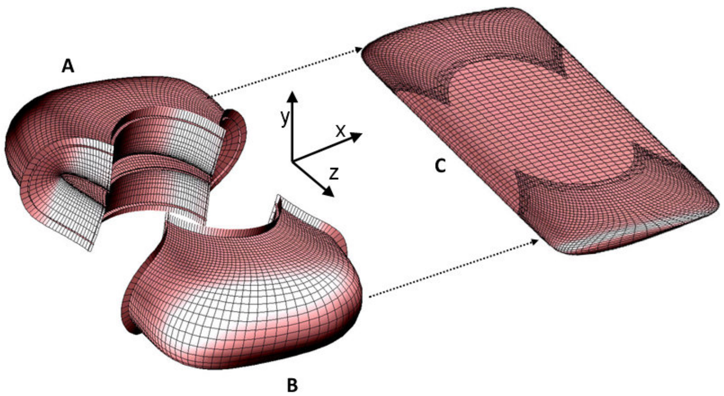
FSI Coupling Issues

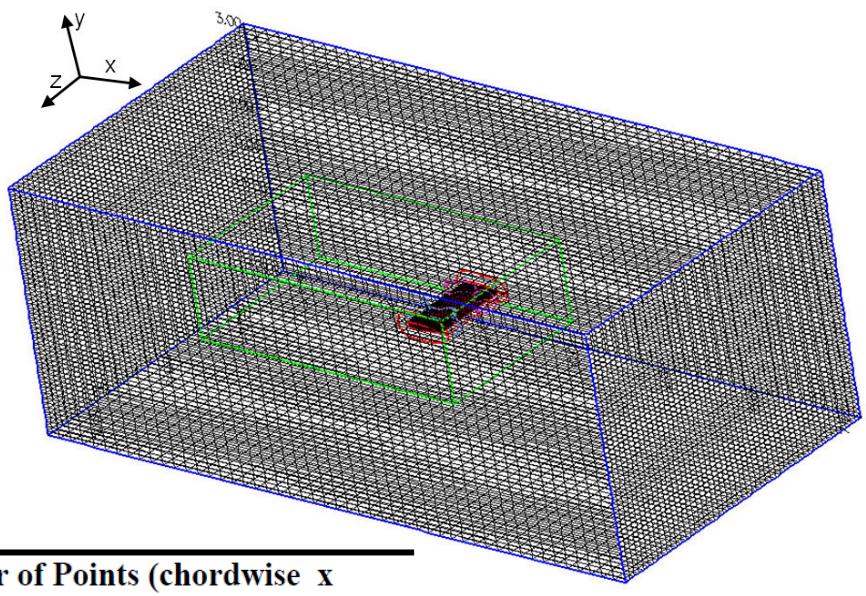
Plunging and passively deforming wing

Rigid Plunging Wing

Comparison of Aerodynamic forces and moments

Experiment	*Heathcote et al. (2008)*
Navier-Stokes	*Young (2005)*
Panel Method	*Young (2005)*
Linear Theory	*Garrick (1936)*





Grid	Number of Points (chordwise x spanwise x normal)
Outer cube	51 x 51x 51
Inner cube	180 x 80 x 90
Wing (C)	172x91x43
Wing Tip (A)	172x61x43
Wing Tip (B)	172x61x43

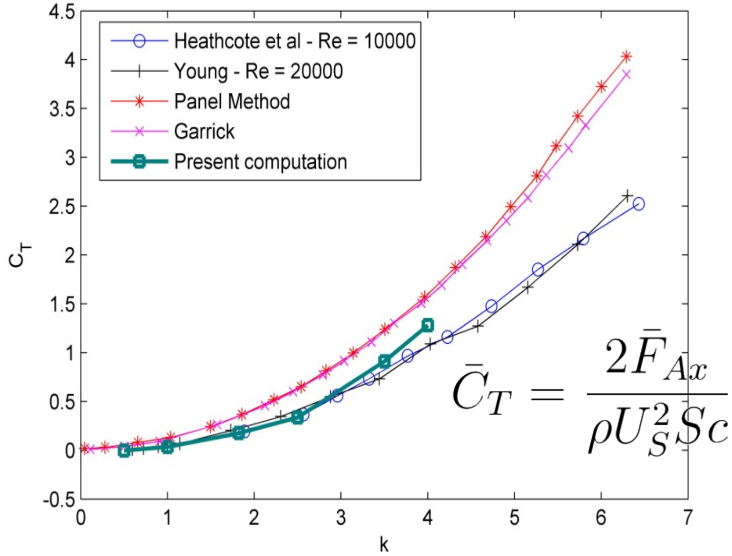
Motion along Y-axis is given by : $h = -h_0 (1 - \cos 2\pi f t)$ with $h_0 = 0.175c$

Reduced frequency, k = 0.5, 1.0, 1.82, 2.5, 3.5 and 4.0

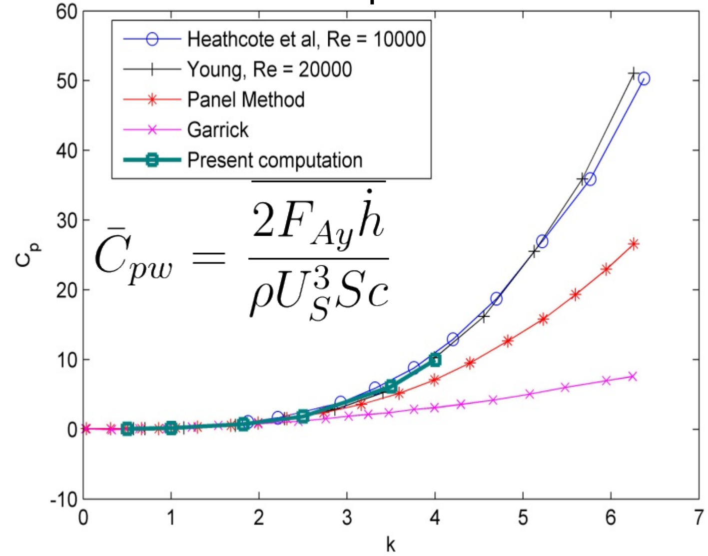
Reynolds number, Re = 10^4

Rigid Plunging Wing

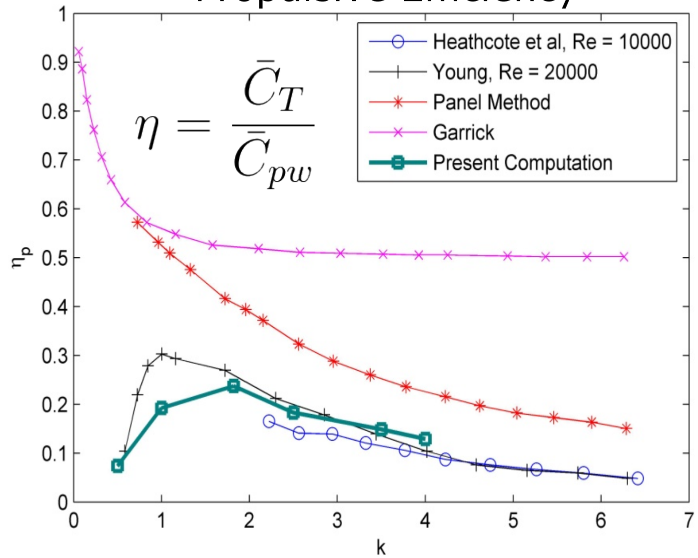
Thrust Coefficient



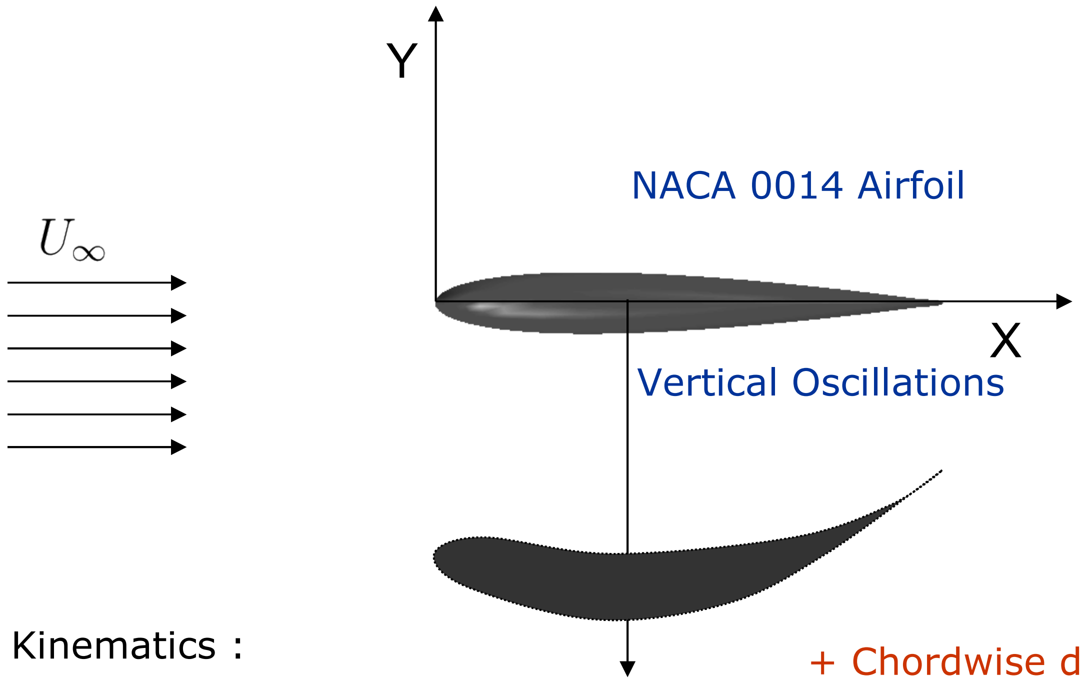
Power Input Coefficient



Propulsive Efficiency



Plunging and Deforming (Active) Airfoil



+ Chordwise deformation

Kinematics :

STATIC: $y(x,t) = y_0(x)$

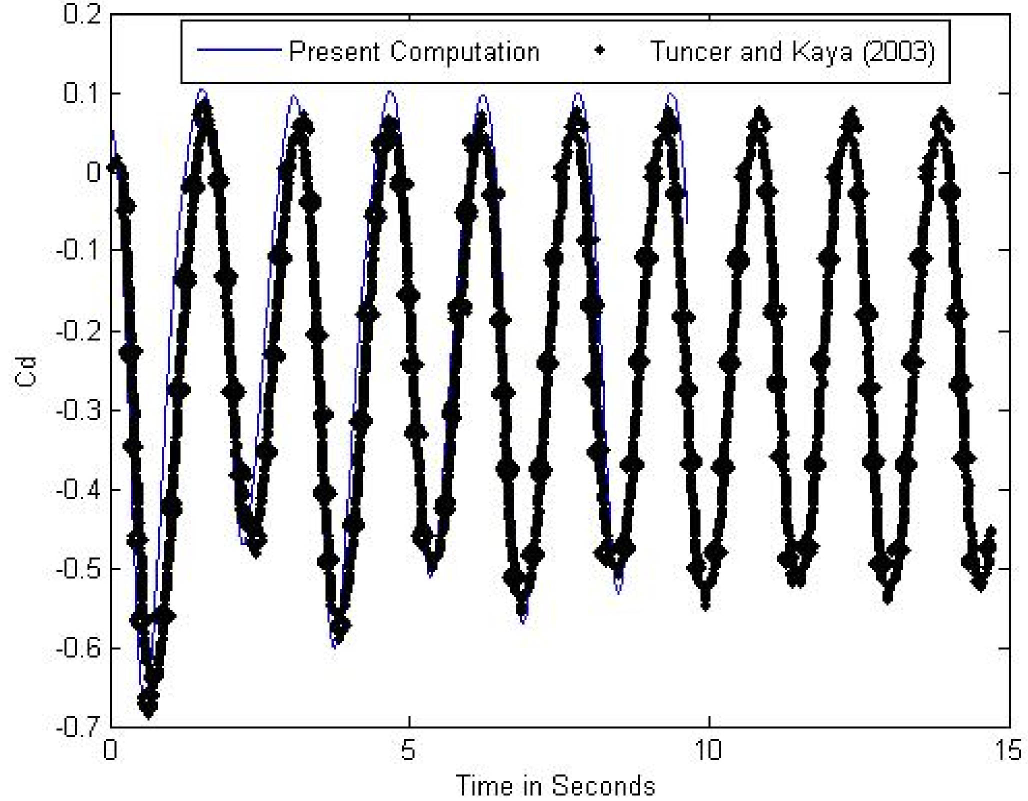
RIGID BODY MOTION: $y(x,t) = y_0(x) + B(1 - cos(\omega t))$

RIGID BODY + DEFORMATION: $y(x,t) = y_0(x) + B(1 - cos(\omega t)) + A(x - x_c)^p cos(\omega t + Qx + \phi)$

1 . Rigid Plunging : *B = 0.4, k=2* , Re=10^4, A =0 - Tuncer and Kaya (2003)

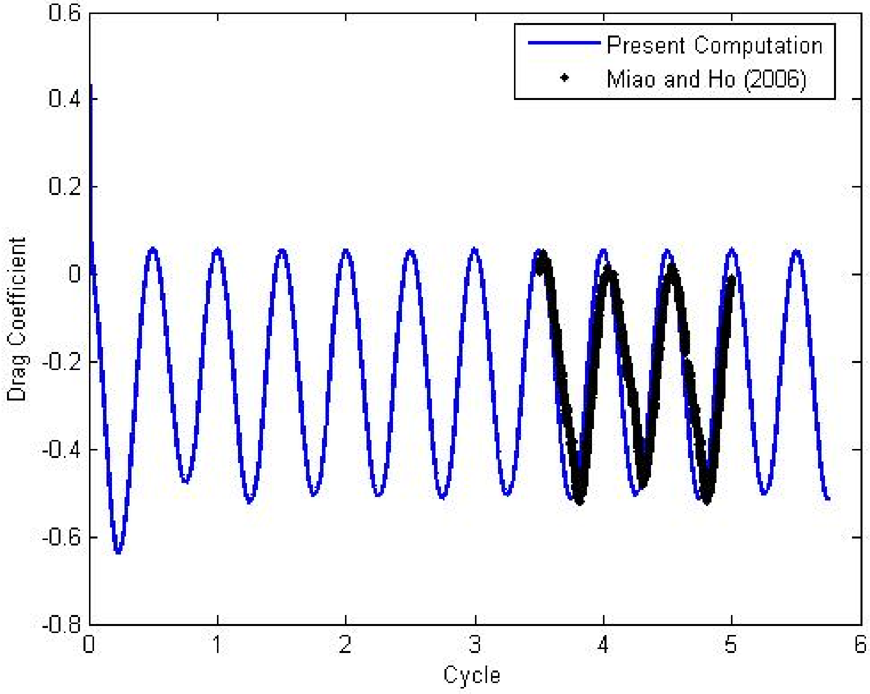
2. Plunging with Deformation : A = 0.3, p = 2, x_c=0, Q=0, ϕ=0, Miao and Ho (2006)

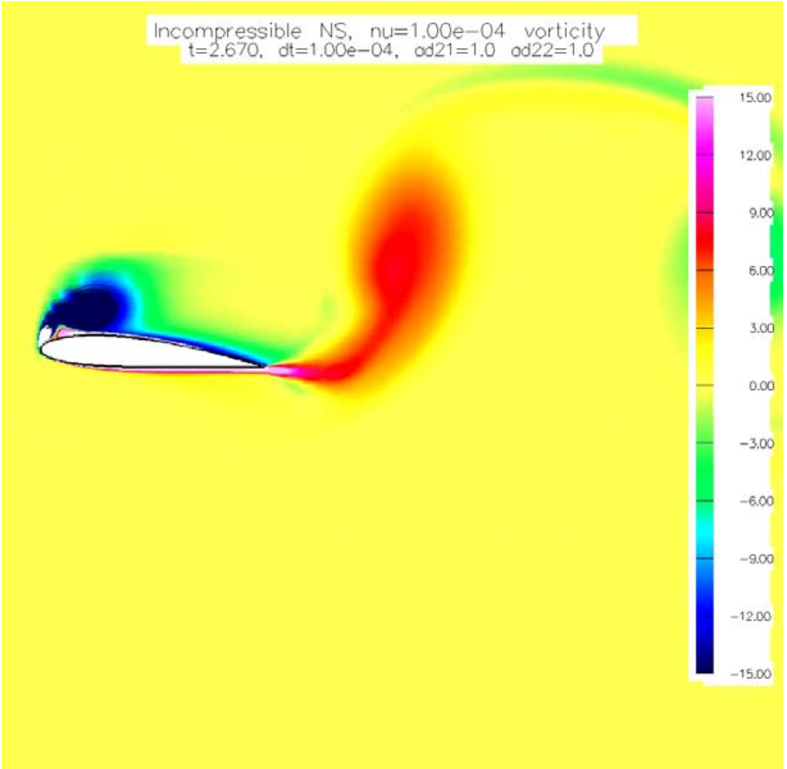
Plunging and Deforming (Active) Airfoil



1 . Rigid Plunging : *B = 0.4, k*=2 , Re=10^4, A =0 - Tuncer and Kaya (2003)

Plunging and Deforming (Active) Airfoil





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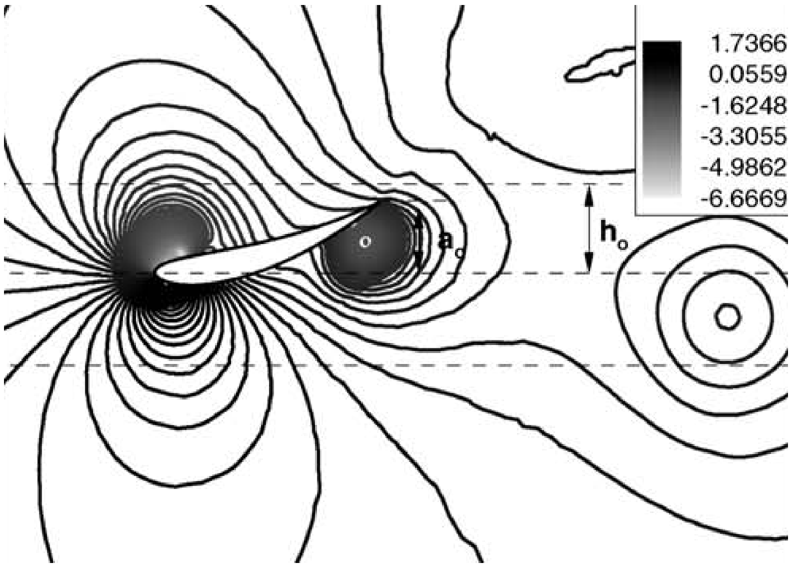
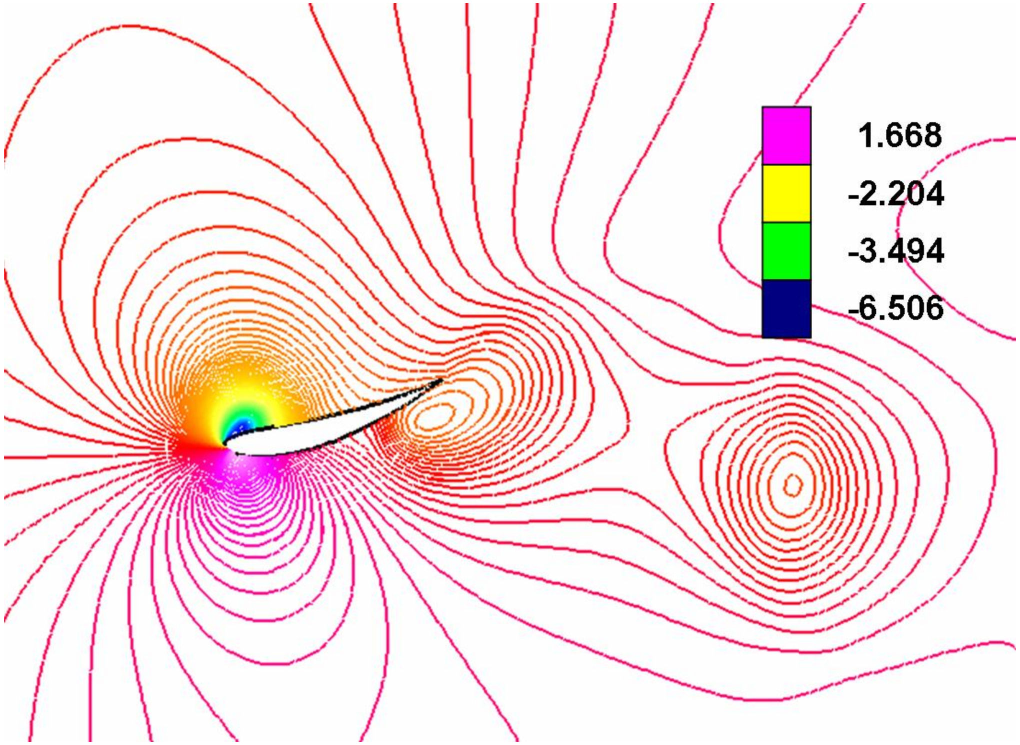
Plunging and Deforming (Active) Airfoil



Miao & Ho (2006)



Present computation

Cp Contours at the mean position

Plunging and Deforming (Passive) Airfoil

Heathcote et al. (2004)

Tang et al. (2007)

Rigid : Length 0.4c, Thickness 0.11c

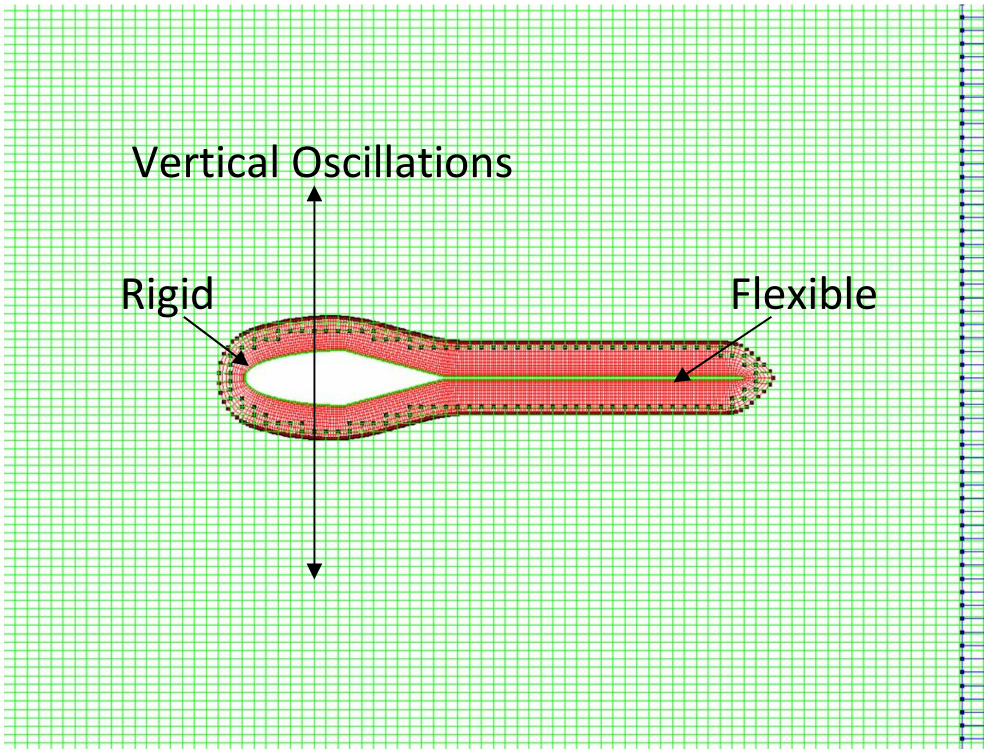
Flex : Length 0.6c, Thickness 0.005c

Re = 9000

$E = 2.05 \times 10^{10} \rho U^2$

$k = fc/U = 1.4$

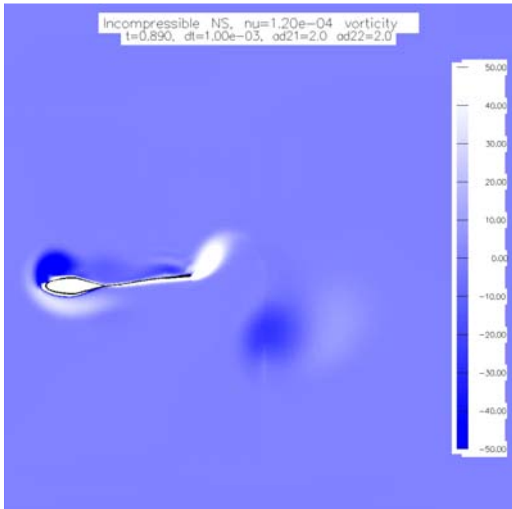
$\rho_b = 7.85\ \rho$



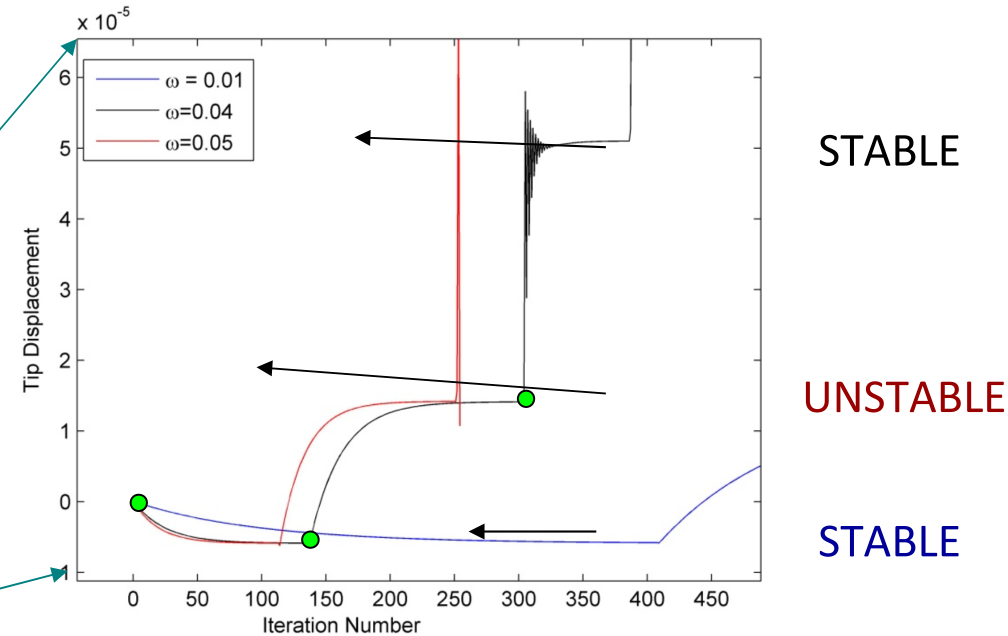
Mesh Size : 500 x 11, 200 x 150

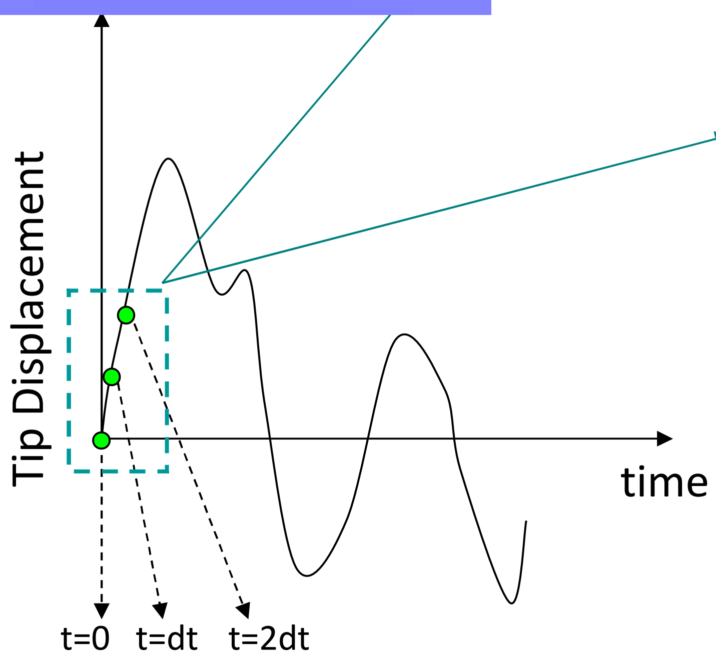
k = reduced frequency , f = frequency, c = beam chord, ρ_b= beam density, ρ = Fluid Density, U = Free-Stream Velocity, E = Modulus of Elasticity

Effect of Time Step, Relaxation and Damping



Tip Displacement With Net Iteration Index for Δt = 1e-3





Rigid : Length 0.4c, Thickness 0.11c

Flex : Length 0.6c, Thickness 0.005c

Re = 9000

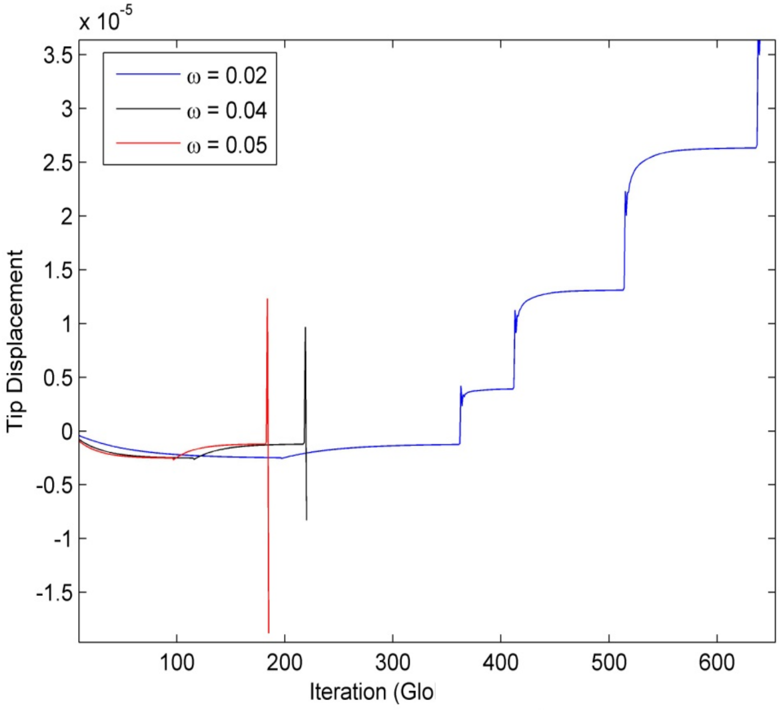
E = $2.05\times10^{10}\rho U^2$

k = fc/U = 1.4

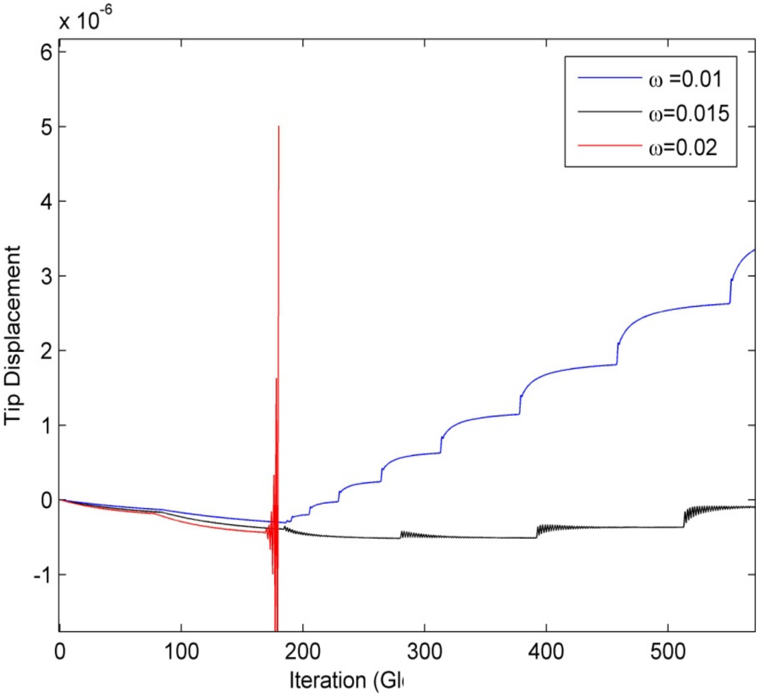
ρ_b = 7.85 ρ

Effect of Time Step, Relaxation and Damping

Δt = 5e-4



Δt = 1e-4



Tip Displacement With Net Iteration Index

Effect of Time Step, Relaxation and Damping

κ \ ω	0.01	0.015	0.0165	0.0169	0.017	0.018	0.0185	0.019	0.02	0.03	0.04	0.05	0.06	0.075
10.0	S	S	S	S	S	S	S	S	S	S	S	U	U	U
15.0	S	S	S	S	S	S	S	S	S	S	S	S	S	U
20.0	S	S	S	S	S	S	S	S	S	S	S	S	S	S/U

Table 1. Stability of the fluid-structure interaction problem with a time step of $\Delta t = 10^{-3}$ for various relaxation and numerical damping coefficients

κ \ ω	0.01	0.015	0.0165	0.0169	0.017	0.018	0.0185	0.019	0.02	0.03	0.04	0.05	0.06	0.075
10.0	S	S	S	S	S	S	S	S	S	S	U	U	U	U
15.0	S	S	S	S	S	S	S	S	S	S	U	U	U	U
20.0	S	S	S	S	S	S	S	S	S	S	S	U	U	U

Table 2. Stability of the fluid-structure interaction problem with a time step of $\Delta t = 5 \times 10^{-4}$ for various relaxation and numerical damping coefficients

κ \ ω	0.01	0.015	0.0165	0.0169	0.017	0.018	0.0185	0.019	0.02	0.03	0.04	0.05	0.06	0.075
10.0	S	S	U	U	U	U	U	U	U	U	U	U	U	U
15.0	S	S	S	U	U	U	U	U	U	U	U	U	U	U
20.0	S	S	S	S	S	S	U	U	U	U	U	U	U	U

Table 3. Stability of the fluid-structure interaction problem with a time step of $\Delta t = 10^{-4}$ for various relaxation and numerical damping coefficients

Flow Induced Deformation of a Beam

Shin et al. (2007)

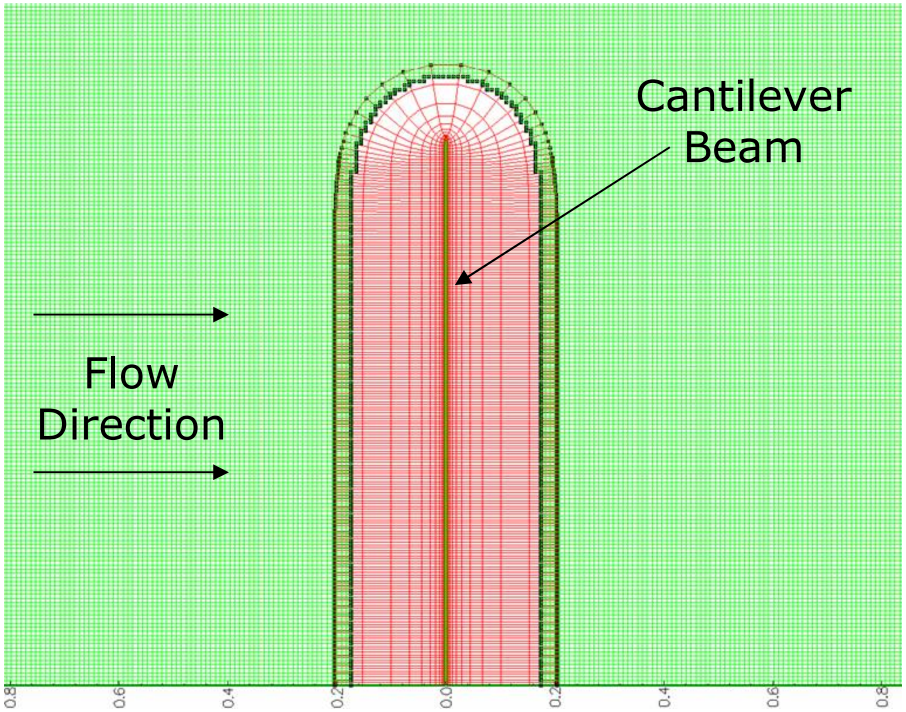
$b/c = 0.003$

$\rho_b/\rho = 6667$

$\Gamma = EI/\rho_b U^2 bc^2 = 2$

$Re = 500$

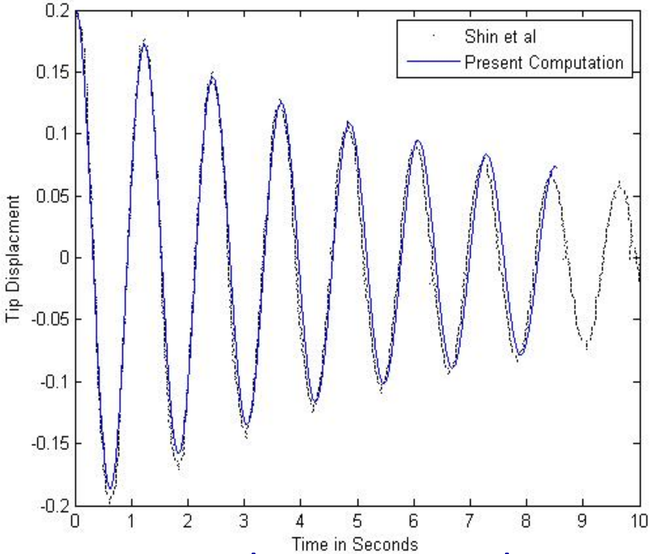
Plate is initially deflected in its first mode for ¼ cycle



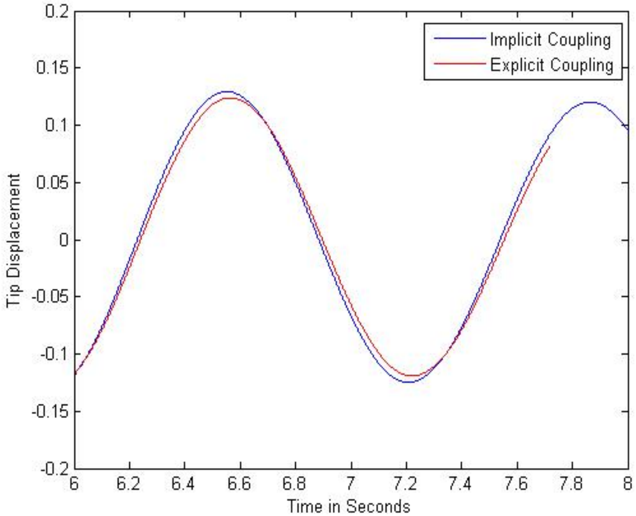
b = beam thickness , c = beam chord, ρ_b= beam density, ρ = Fluid Density, U = Free-Stream Velocity

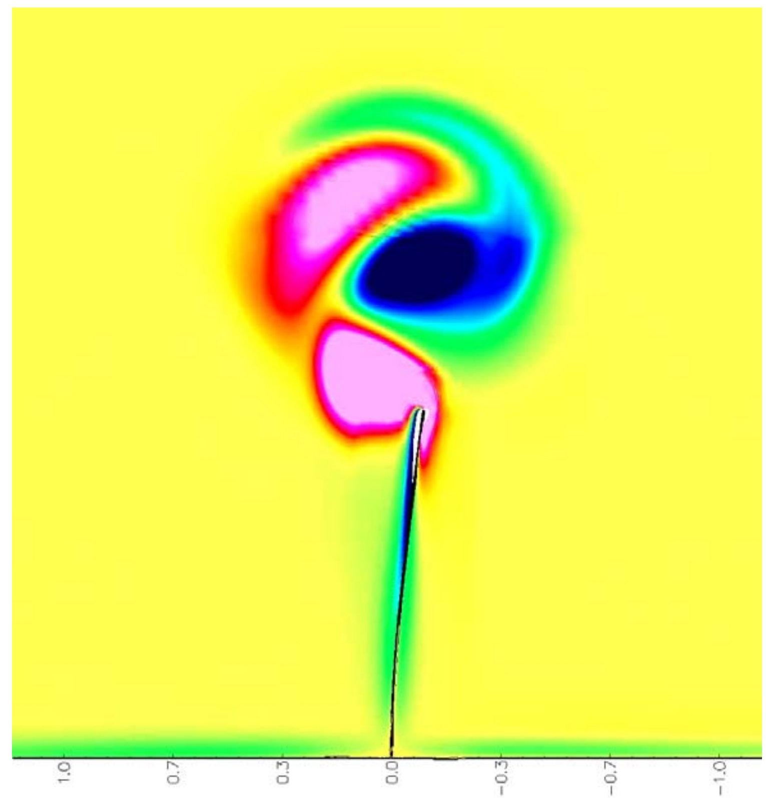
E = Modulus of Elasticity, I = moment of Inertia

Flow Induced Deformation of a Beam



Tip Displacement with Time





Vorticity Contours

Explicit vs Implicit coupling

Plunging and Deforming Wing

Heathcote et al. (2008) - Experiments

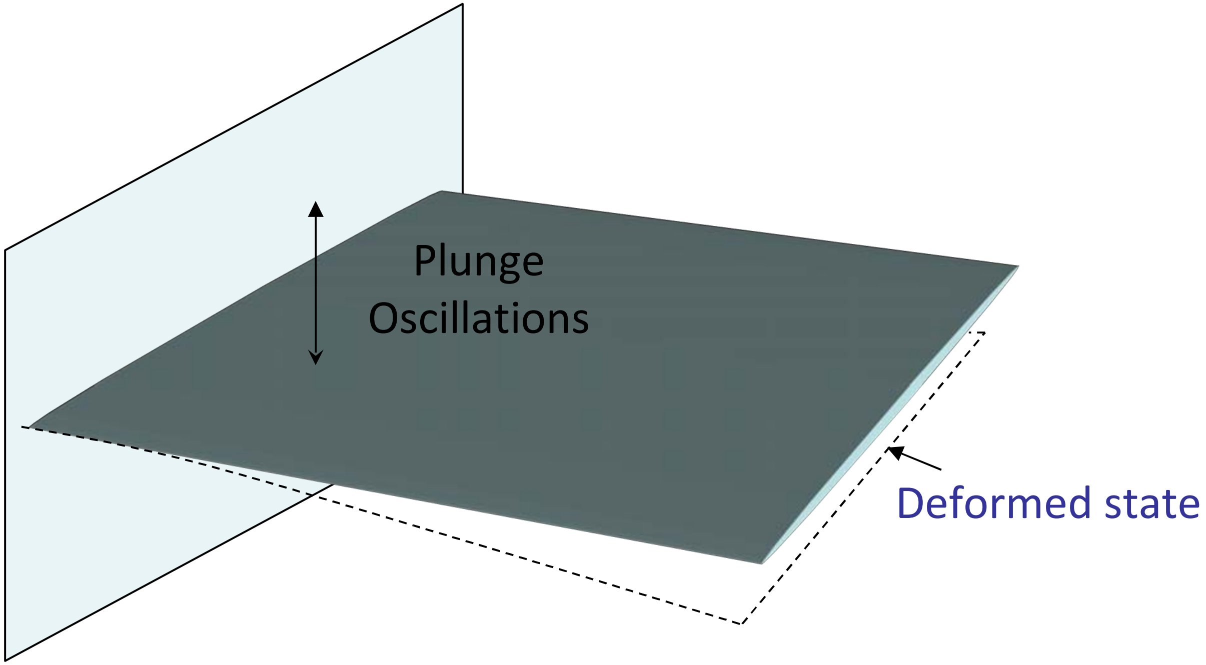
Aono et al. (2009) - Computations

Re = 10000

E = 200 Gpa

k = fc/U = 1.82

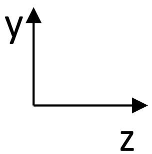
ρ_b = 7.85 ρ (Steel Plate)

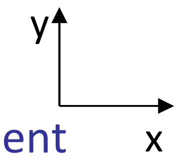


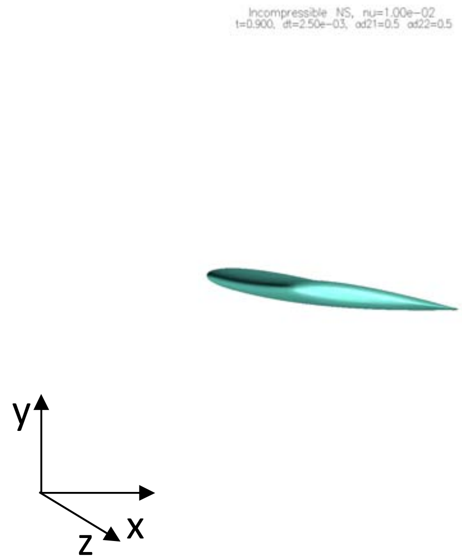
Deformation along the Span-wise direction

Typically 20 Inner Iterations (Correction Steps) with a $\Delta t = 2.5 \times 10^{-3}$ s

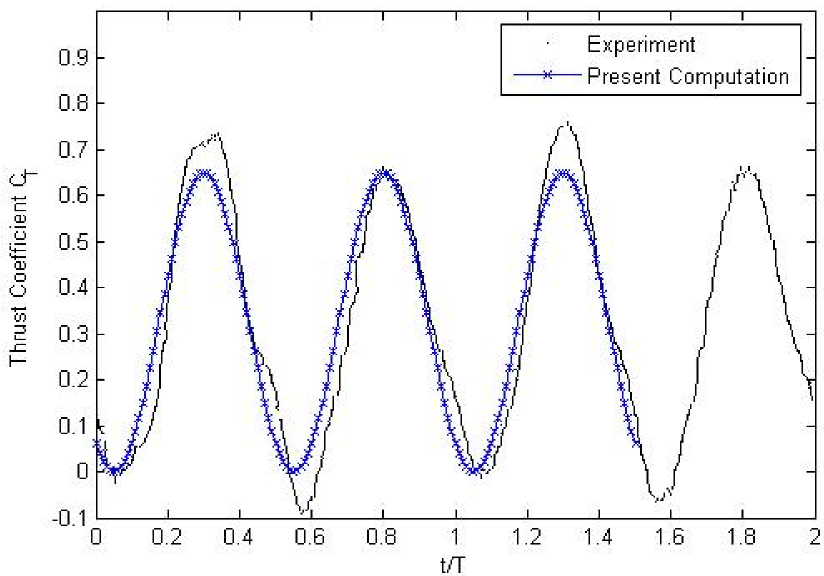
Plunging and Deforming Wing

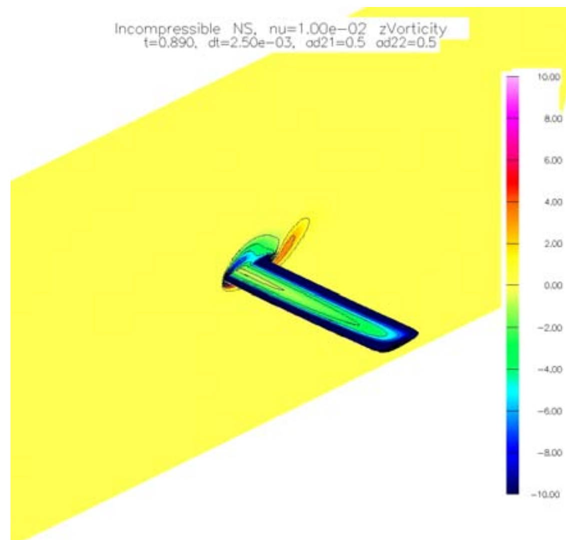






Thrust Coefficient





Concluding Remarks

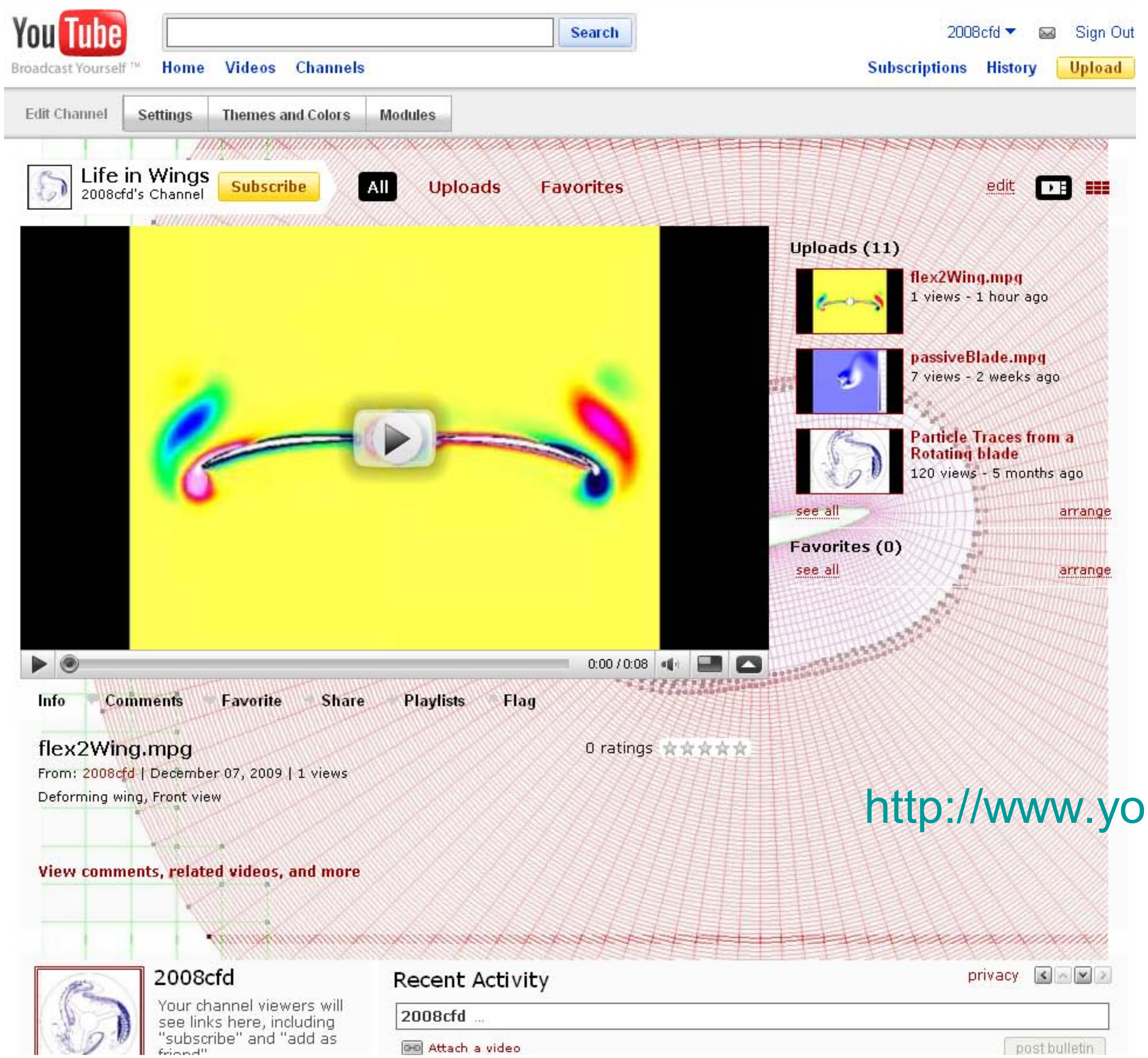
A computational framework has been developed to couple fluid dynamics, rigid body dynamics, and structural dynamics

→ Developed a partitioned coupling approach for fluid-structure interaction problems

→ Importance of relaxation for partitioned coupling approaches

→ Coupling an external structural solver with cgins – under progress

Access my Channel @ youtube



http://www.youtube.com/user/2008cfd