Delivering Breakthrough Solutions

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A Coupled Overset Vorticity Transport and Euler Solver for Vortex-Dominated Flows

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Supported by US Army Aeroflightdynamics Directorate (AFDD)

Outline of Presentation

- **Introduction and Motivation**
- Formulation of Vorticity Transport Solver
- Formulation of Compressible Euler Solver
- Overset Coupling Methodology
- Results and Discussion
- Summary and Conclusions

Introduction and Motivation

- CFD has seen significant progress in recent years in modeling the underlying flow physics and complex aerodynamic behavior associated with vortex-dominated flows
- Such flows include high-lift configuration, helicopter rotor tip vortex capture and preservation, prediction of rotor wash and blade-vortex interactions, etc
- Accurate simulation of these flows requires a sufficiently resolved high-fidelity calculation in which vortical structures are captured and preserved for several blade revolutions
- Accomplishing this using a 2nd-order primitive-variable formulation of the Navier-Stokes equations can be difficult, as these formulations are often prone to excessive numerical dissipation of vortical structures

Introduction and Motivation

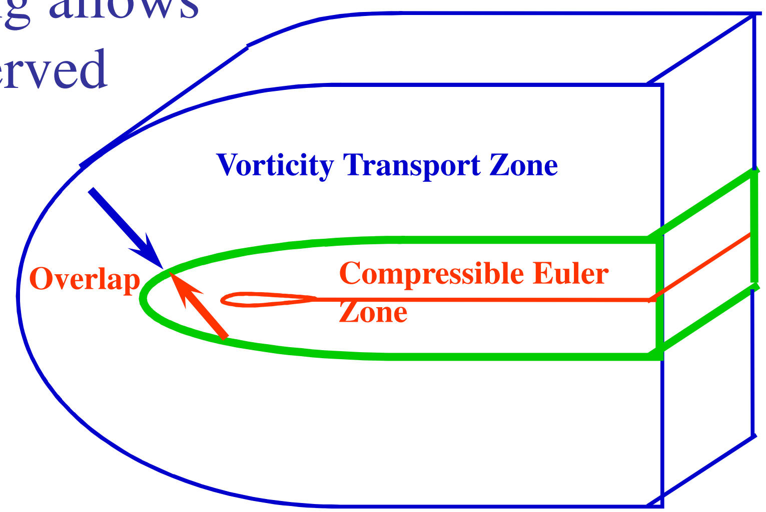
- Velocity-vorticity formulations of the Navier-Stokes equations offer several advantages over primitive-variable formulations and have been the recent focus of major research efforts

- Since the Eulerian Vorticity Transport (EVT) formulation deals with vorticity as the fundamental conserved quantity, there is inherently less smearing and dissipation of vortical structures than in a comparable 2nd-order Navier-Stokes solution

- Additionally, there are fewer primary equations in the EVT solution process and the equations themselves are simpler and involve fewer operations than in Navier-Stokes solutions

Objectives

- Provide a novel CFD simulation capability for accurate and efficient simulation of vortex-dominated flows
- Enable effective coupling of compressible Euler solver in near-body region with incompressible adaptive Cartesian EVT solver in off-body wake region using overset grid technology
- Vorticity field originates at solid surfaces in near-body region and is transported into wake region employing off-body EVT solver
- Adaptive mesh refinement and coarsening allows vortical structures to be efficiently preserved with high-fidelity in wake region



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Mathematical Formulation of EVT

Consider the Navier-Stokes equations written in velocity-vorticity form:

$$\frac{\partial \vec{\omega}}{\partial t} + (\vec{V} \cdot \nabla)\vec{\omega} - (\vec{\omega} \cdot \nabla)\vec{V} = \nu \nabla^2 \vec{\omega}$$

Velocity and vorticity are related through the Biot-Savart relationship:

$$\nabla^2 \vec{V} = -\nabla \times \vec{\omega}$$

which has the solution:

$$\vec{V}(\vec{x}) \approx \int_{\forall} K(\vec{x}, \vec{y}) \times \vec{\omega}(\vec{y}) d\vec{y} \qquad \vec{K}(\vec{x}, \vec{y}) = -\frac{1}{4\pi} \frac{(\vec{x} - \vec{y})}{|\vec{x} - \vec{y}|^3}$$

Pressure computed from: $\nabla^2 p = -\rho\left[tr\left(\nabla \vec{V} \cdot \nabla \vec{V}\right)\right]$

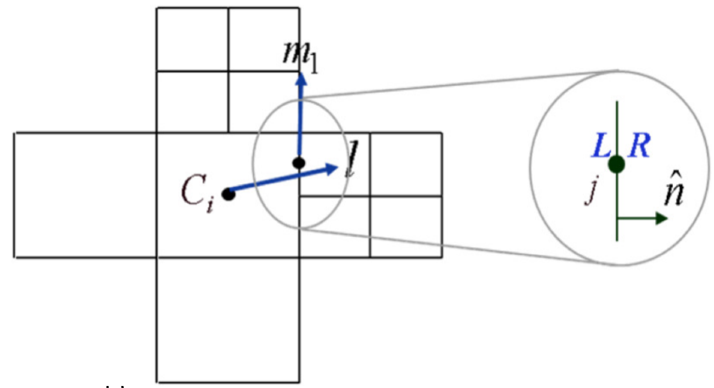
We employ two approaches here for velocity solution:

1) Multigrid Poisson solution
2) Treecode method for integral form (Lindsay & Krasny, JCP 2001)

Mathematical Formulation of EVT

Integrating over cell C_i:

$$\frac{\partial \vec{\omega}_i}{\partial t}\mathcal{V}_i + \sum_j \left(\vec{F}_I - \vec{F}_V\right) - \left[\left(\vec{\omega}_i \cdot \nabla\right)\vec{V}_i\right]\mathcal{V}_i = 0$$



where

$$\vec{F}_I \cong \frac{A_j}{2}\left\{\vec{\omega}_L\left[\vec{V}_L \cdot \hat{n} + \frac{1}{2}\left|\left(\vec{V}_L + \vec{V}_R\right)\cdot \hat{n}\right|\right] + \vec{\omega}_R\left[\vec{V}_R \cdot \hat{n} - \frac{1}{2}\left|\left(\vec{V}_L + \vec{V}_R\right)\cdot \hat{n}\right|\right]\right\}$$ (Rusanov flux)

and $\vec{F}_V \cong \nu A_j \left(\nabla \tilde{\omega} \cdot \hat{n}\right)$

where $\nabla \tilde{\omega} \cdot \hat{m}_{1,2} = \frac{1}{2}\left(\nabla \omega_L \cdot \hat{m}_{1,2} + \nabla \omega_R \cdot \hat{m}_{1,2}\right)$ $\nabla \tilde{\omega} \cdot \hat{l} = \frac{\omega_R - \omega_L}{\left|\vec{r}_R - \vec{r}_L\right|}$

Employing the 2nd-order explicit Runge-Kutta scheme , we obtain

$$R\left(\vec{\omega}_i\right) = -\frac{1}{\mathcal{V}_i}\sum_j \left(\vec{F}_I - \vec{F}_V\right) + \left(\vec{\omega}_i \cdot \nabla \vec{V}_i\right)$$

$$\vec{\omega}^{(1)} = \vec{\omega}^n + \Delta t R\left(\vec{\omega}^n\right)$$

$$\vec{\omega}^{n+1} = \frac{1}{2}\left(\vec{\omega}^{(1)} + \vec{\omega}^n + \Delta t R\left(\vec{\omega}^{(1)}\right)\right)$$

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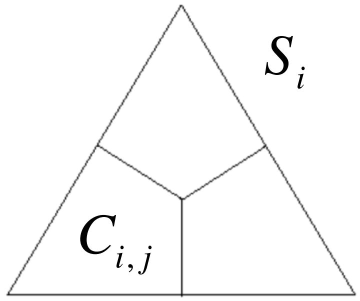
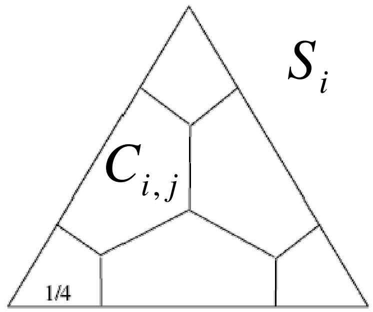
Formulation of Compressible Euler Solver

Consider $$\frac{\partial Q}{\partial t} + \frac{\partial f(Q)}{\partial x} + \frac{\partial g(Q)}{\partial y} = 0,$$

with initial condition $Q(x,y,0) = Q_0(x,y)$, and appropriate BCs on domain discretized into triangular cells



where $$Q = \begin{Bmatrix} \rho \\ \rho u \\ \rho v \\ E \end{Bmatrix} \quad f = \begin{Bmatrix} \rho u \\ \rho u^2 + p \\ \rho uv \\ u(E+p) \end{Bmatrix} \quad g = \begin{Bmatrix} \rho v \\ \rho uv \\ \rho v^2 + p \\ v(E+p) \end{Bmatrix}$$



where $E=p/(\gamma-1)+\rho(u^2+v^2)/2$ is the total energy and $\gamma=1.4$

We employ the spectral volume method (Z. J. Wang, JCP 2002), in which the triangular grid cells are further subdivided into control volumes

Formulation of Compressible Euler Solver

Integrating the conservation law on $C_{i,j}$, we obtain

$$\frac{d\overline{Q}_{i,j}}{dt} = R_{i,j}\left(\overline{Q}\right) = -\frac{1}{V_{i,j}}\sum_{r=1}^{K}\int_{A_r}(F \cdot \hat{n})dA$$

where the solution is reconstructed from the cell-averages using Lagrange-like polynomials of the form

$$Q(x,y) = \sum_{j=1}^{N} L_j(x,y)\overline{Q}_{i,j}, \qquad \text{where} \qquad \int_{C_{i,j}} L_k(x,y)\,dV = V_{i,j}\delta_{jk}$$

The surface integration on each face can be performed using quadrature rules, or a quadrature free approach

We employ the explicit Runge-Kutta scheme for time integration

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Preliminaries:

- **Overset grid assembly and interpolation between solvers provided by SUGGAR/DiRTlib libraries (Ralph Noack – Penn State ARL)**
- **EVT solver requires $(\vec{V}, \vec{\omega})$ values interpolated from Euler solver, while Euler solver requires $(p, \rho, \vec{V})$ values interpolated from EVT solver**

- **Vorticity field must be computed in Euler domain at every step**
- **Pressure field must be computed in EVT domain at every step**

$$\nabla^2 p = -\rho\left[tr\left(\nabla\vec{V} \cdot \nabla\vec{V}\right)\right]$$

- **Density computed in EVT based on isentropic assumption** $\frac{p}{\rho^\gamma} = Const$

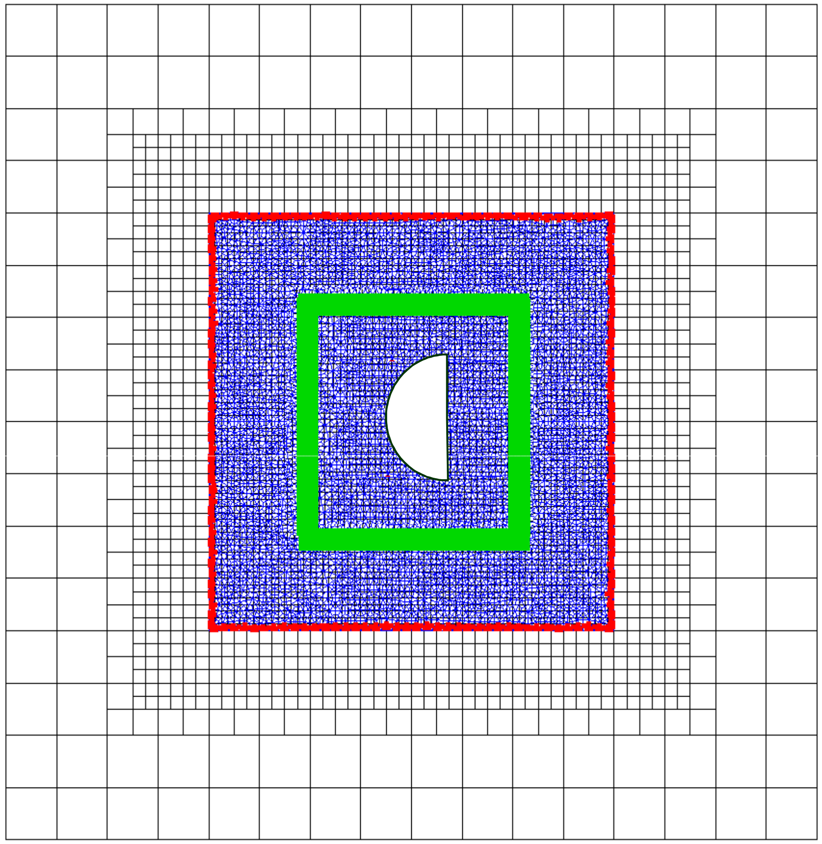
- Velocity field computed in EVT based on two different approaches
  1) Multigrid Poisson solution
  2) Treecode method for integral form (Lindsay & Krasny, JCP 2001)

EVT Velocity Fromulation

Method 1: Multigrid Poisson solution

$$\nabla^2 \vec{V} = -\nabla \times \vec{\omega}$$

Poisson equation solved in EVT domain using velocity and vorticity interpolated to EVT receptor cells (green) as BCs



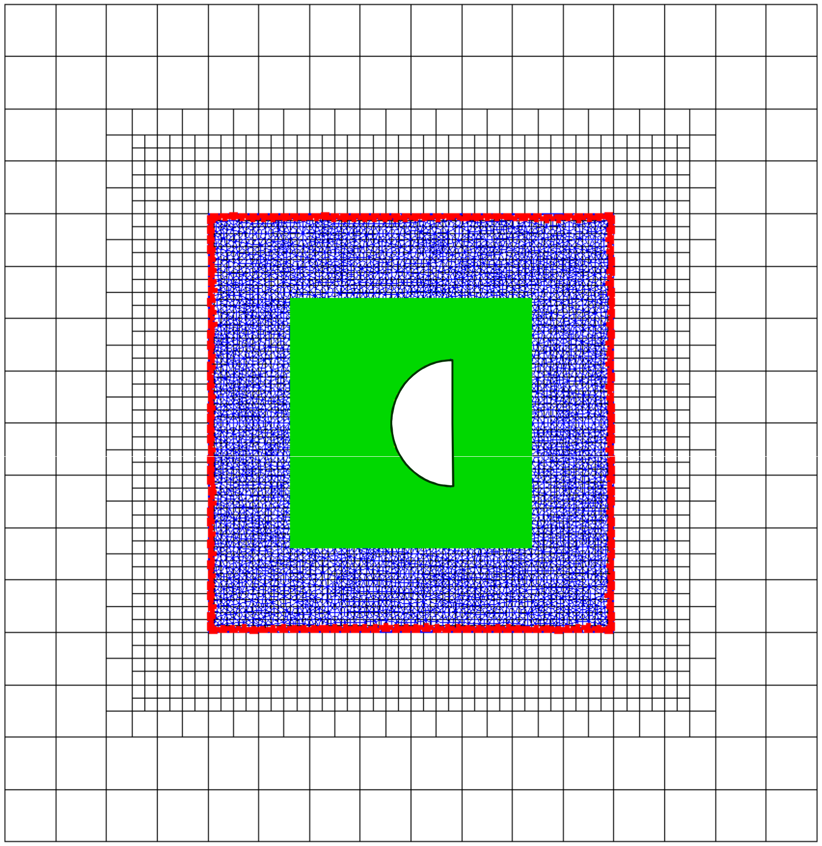
Euler receptor cells
EVT receptor cells

Method 2: Treecode method

$$\vec{V}_R(\vec{x}) \approx \int_{V} K(\vec{x},\vec{y}) \times \vec{\omega}(\vec{y}) d\vec{y}$$

EVT utilizes integral form: velocity is influenced by entire vorticity field (i.e. full Euler domain also)

EVT "out" cells retained as "fringe" cells to store Euler vorticity values



Euler receptor cells
EVT receptor cells

Helmholtz decomposition to include irrotational flow introduced by body

$$\vec{V} = \vec{V}_R + \nabla \phi$$

Continuity then gives:

$$\nabla^2 \phi = -\nabla \cdot \vec{V}_R \quad \text{where} \quad \phi_\infty(\vec{x}) = \vec{V}_\infty \cdot \vec{x}$$

$$(\partial \phi / \partial n)_{WALL} = 0$$

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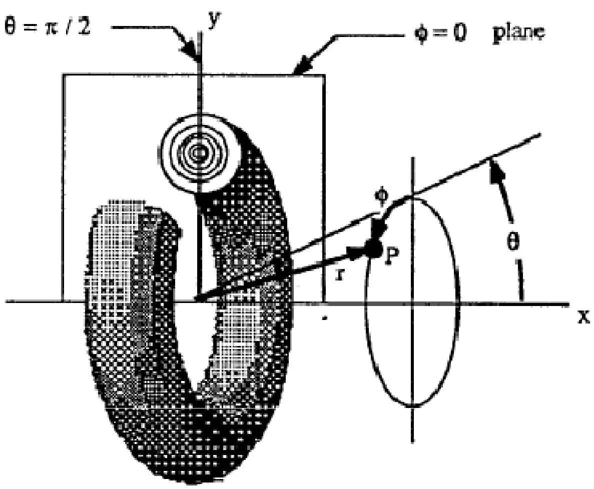
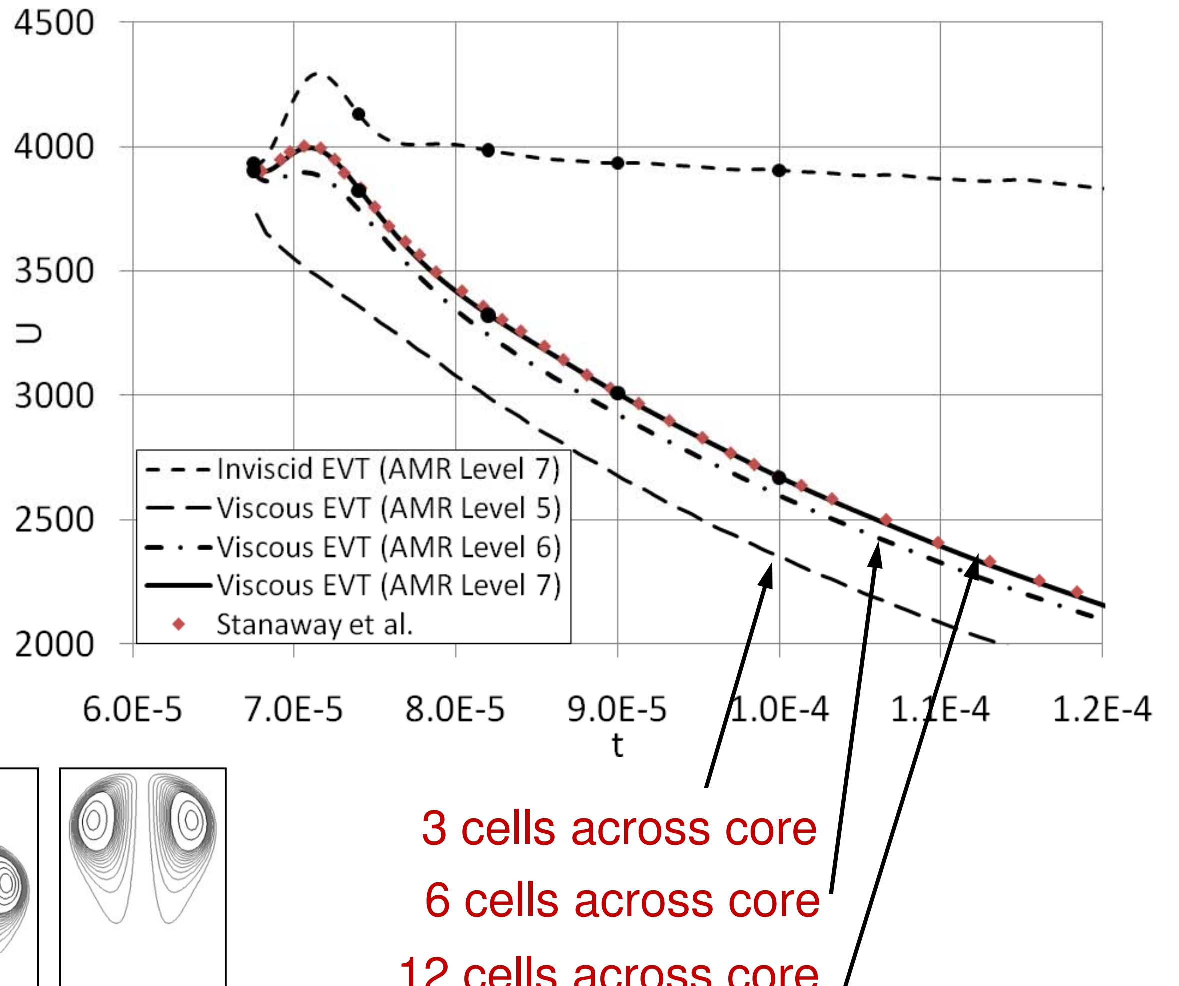
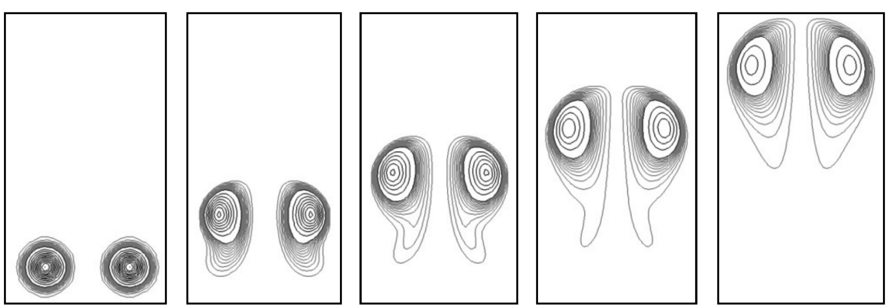
Validation of Stand-alone EVT

Viscous Ring vortex propagation

S. K. Stanaway, B. J. Cantwell, and P. R. Spalart, Navier-Stokes simulations of axisymmetric vortex rings, AIAA Technical Paper, AIAA-88-0318, 1988.



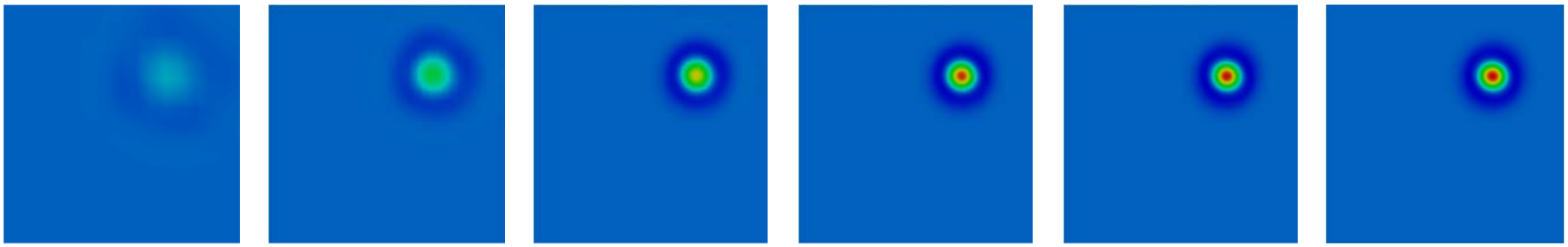




Efficiency/Accuracy of stand-alone EVT/Euler solvers

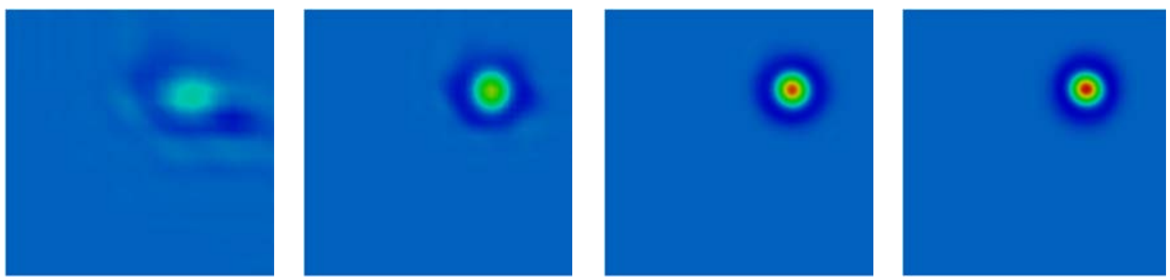
- Vortex propagation with V=(0.1,0.1) until t=2

EVT



256 cells | 1,024 cells | 4,096 cells | 16,384 cells | 65,536 cells | 262,144 cells

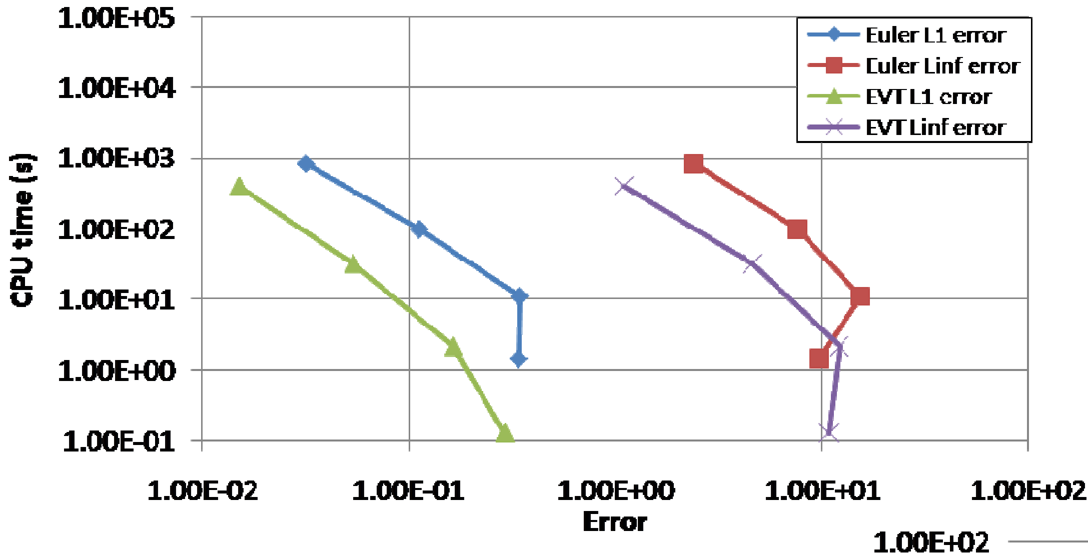
Euler



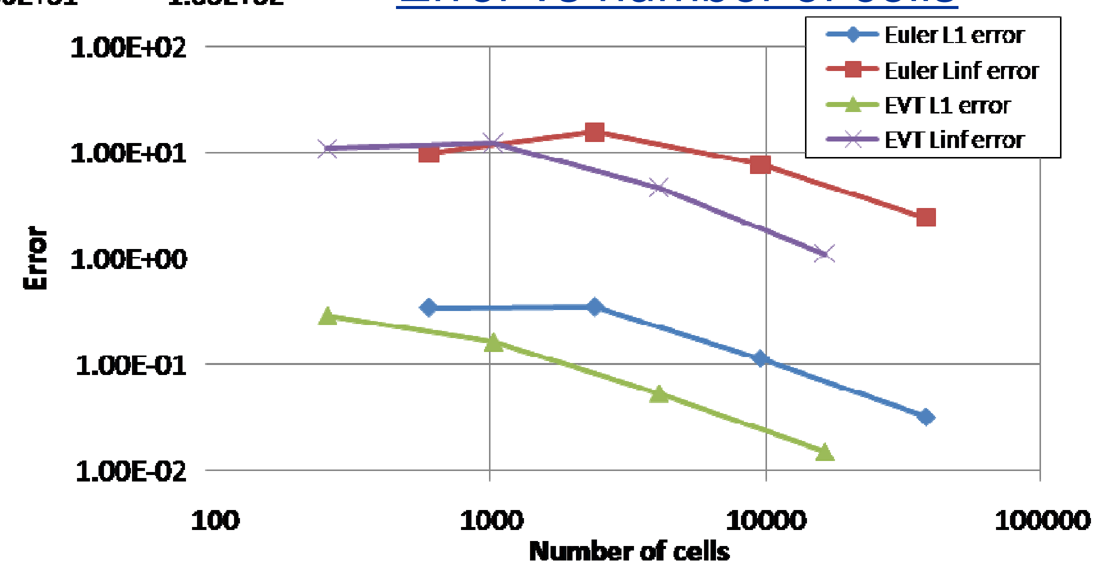
2,400 cells | 9,600 cells | 38,400 cells | 153,600 cells

Efficiency/Accuracy studies of EVT and Euler solvers

Relative cost to achieve desired error

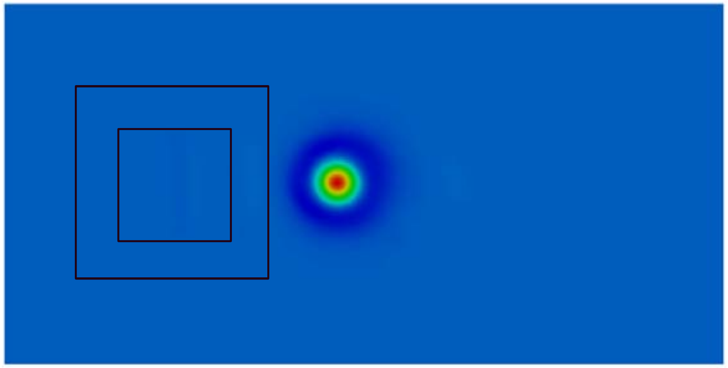


Error vs number of cells

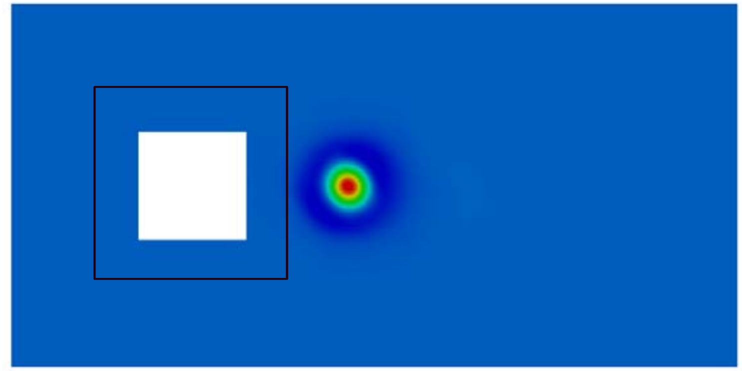


- Euler is approximately one order-of-magnitude more expensive than EVT to achieve desired error!

Results for coupled EVT-Euler vortex propagation at Mach=0.2

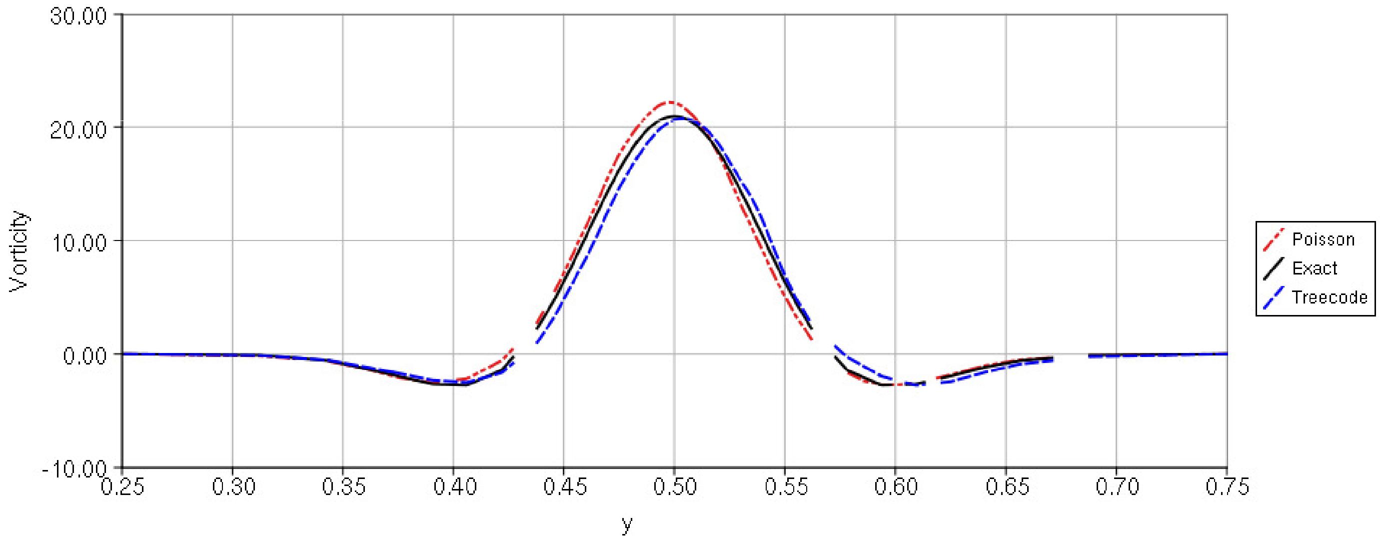


Vorticity



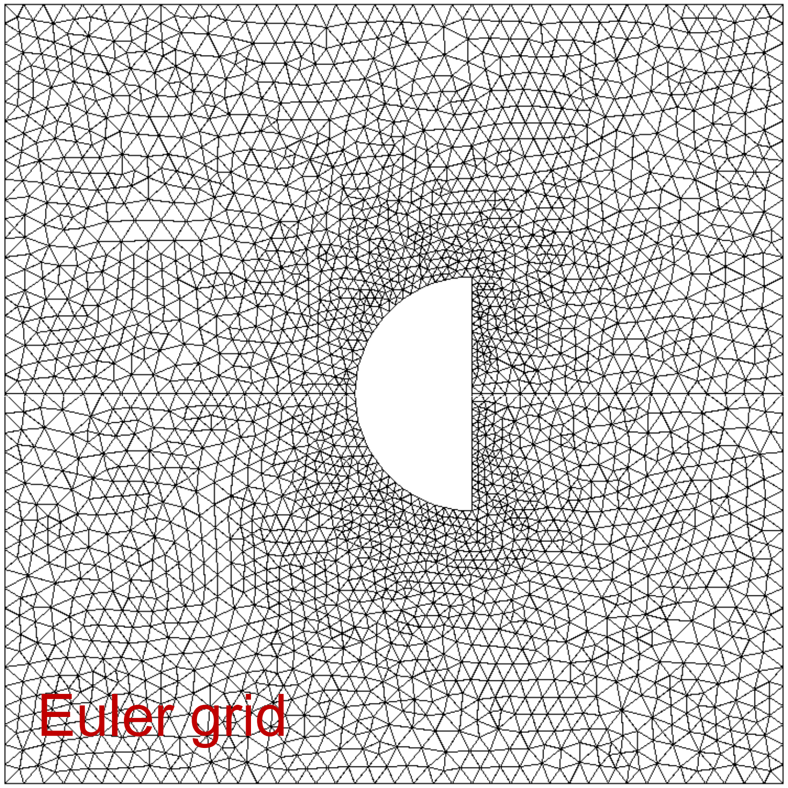
Treecode approach

Poisson approach

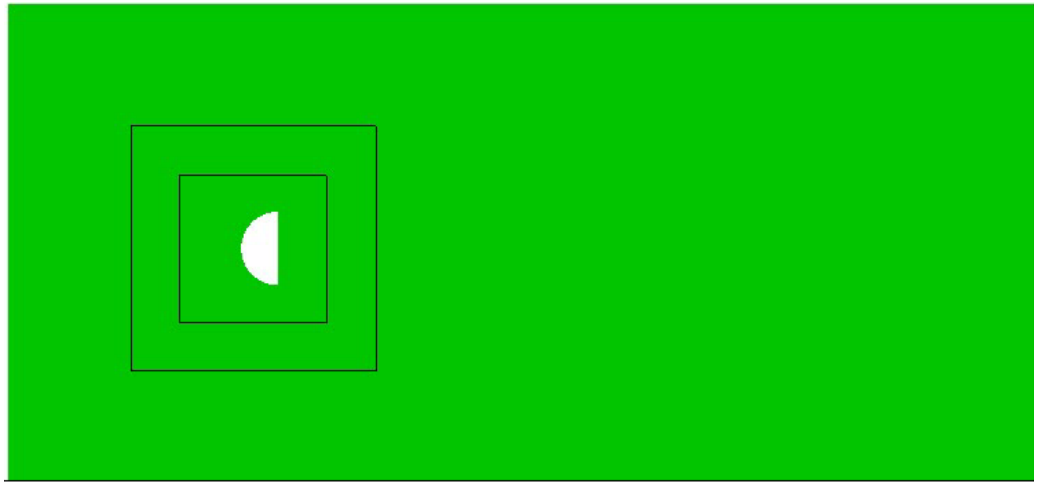


Vorticity in *y* direction at *x*=0.43, *t*=2.1486 (~10 cells across core)

Mach=0.2 flow over half cylinder

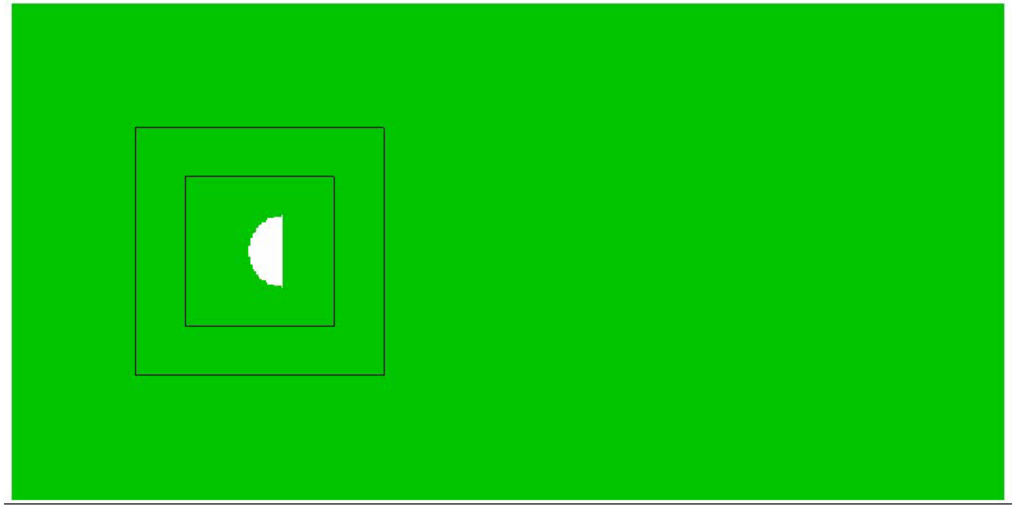


Mach=0.2 flow over half cylinder (vorticity contours)



Poisson approach

Treecode approach



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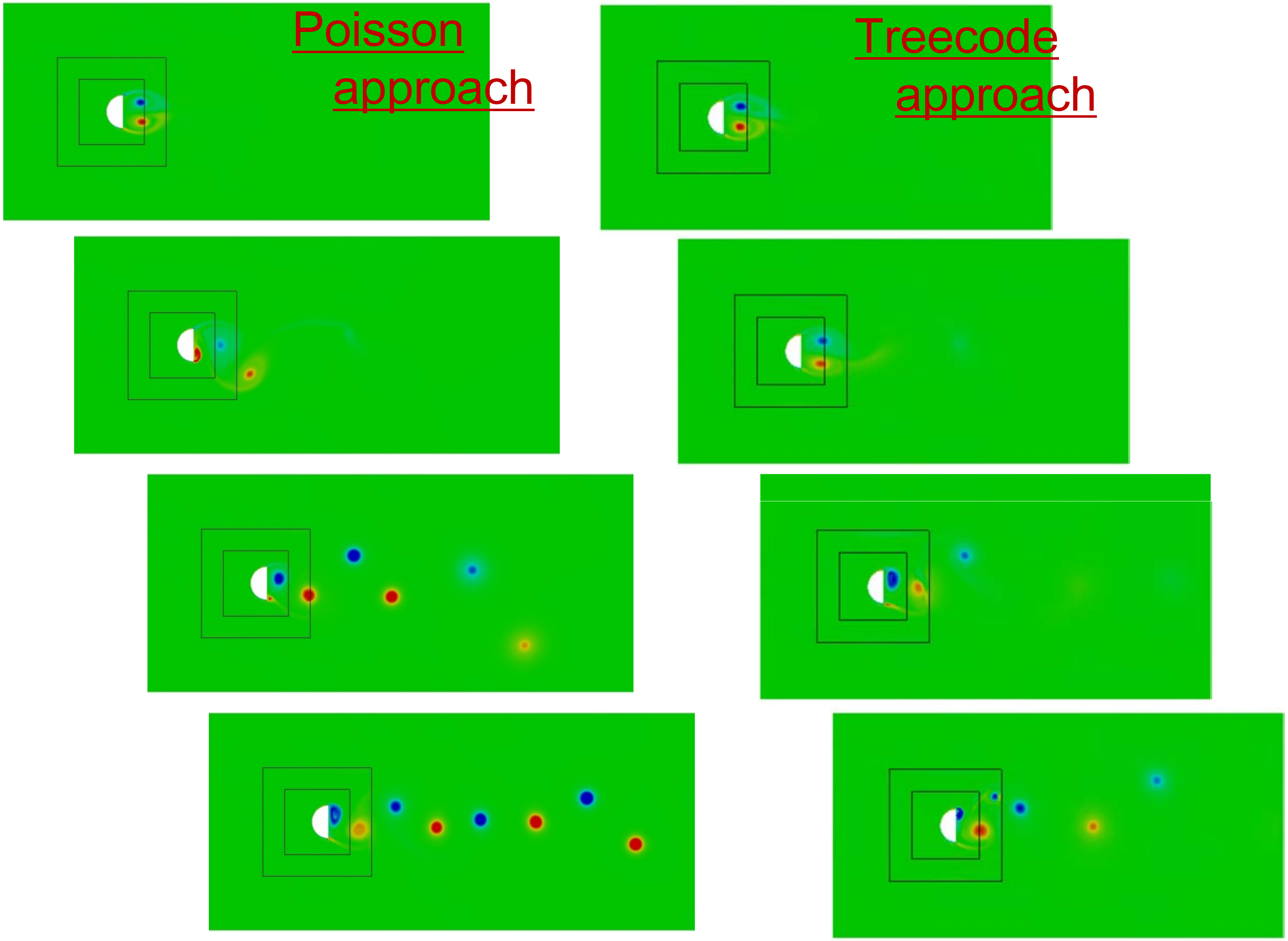
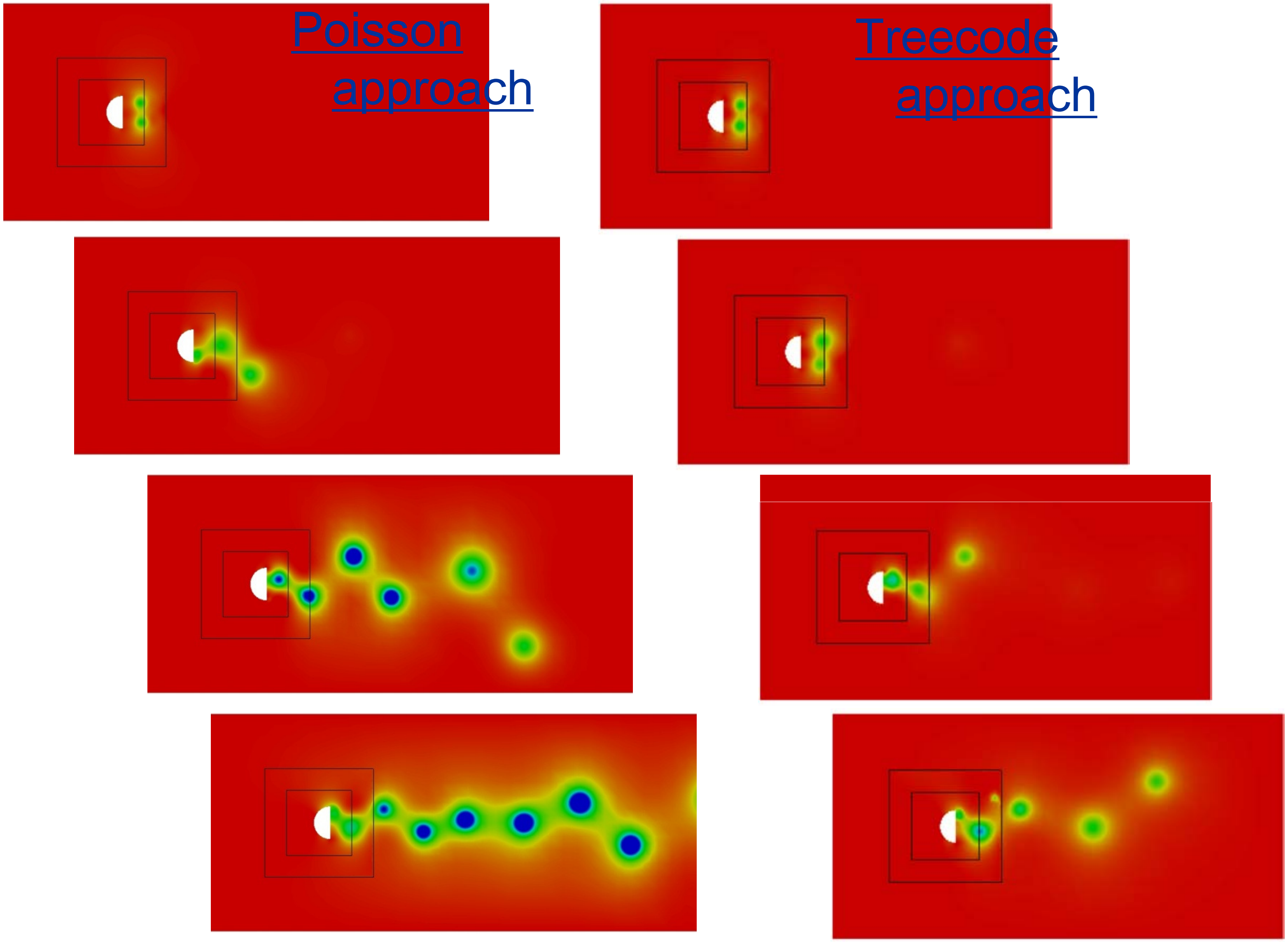
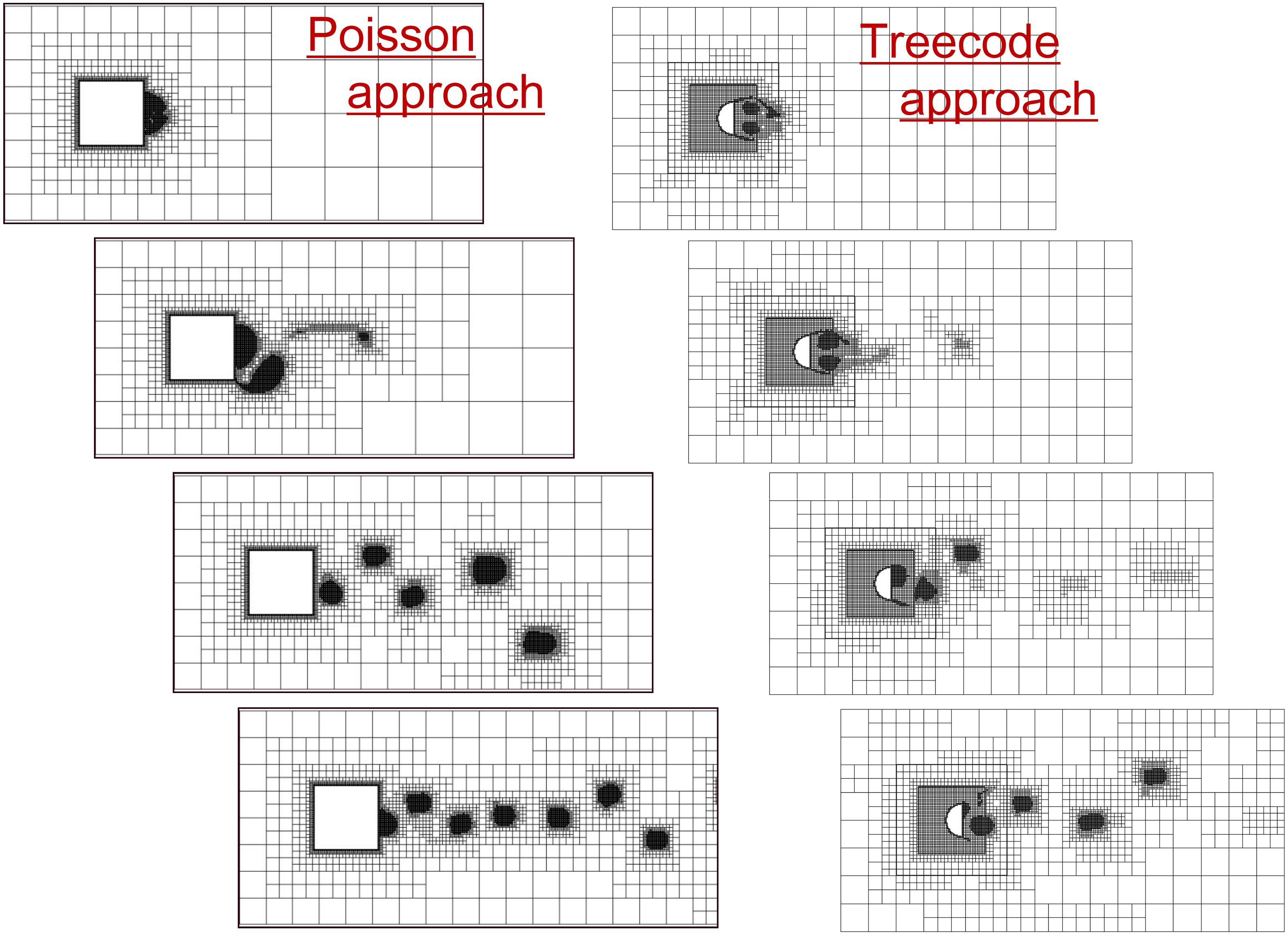
Mach=0.2 flow over half cylinder (vorticity contours)



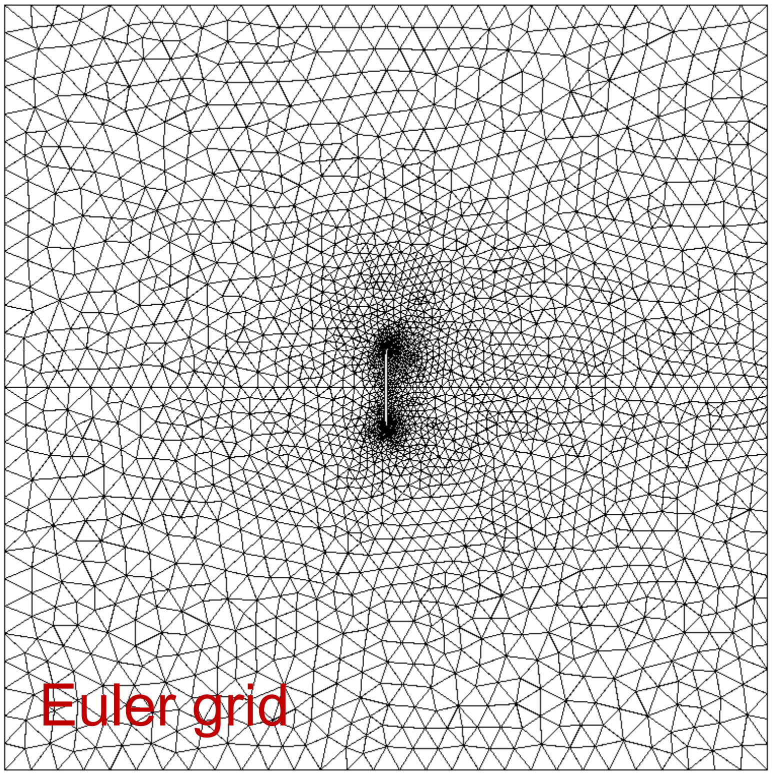
Mach=0.2 flow over half cylinder (pressure contours)

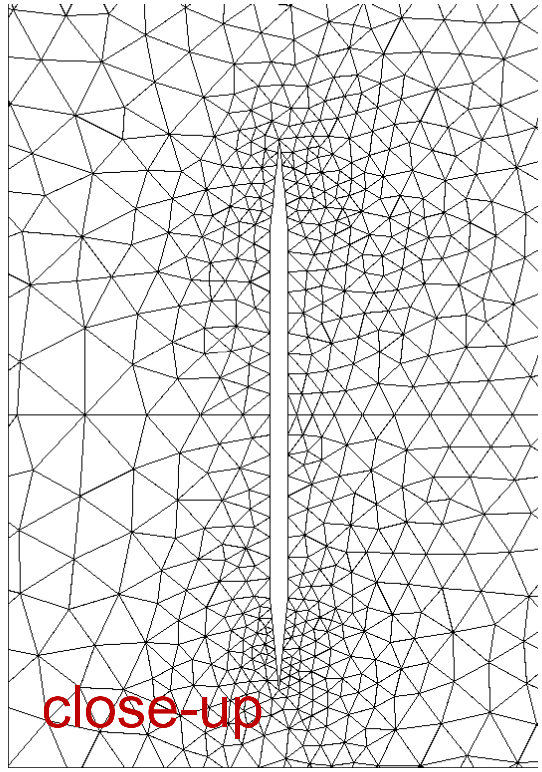


Mach=0.2 flow over half cylinder (grids)

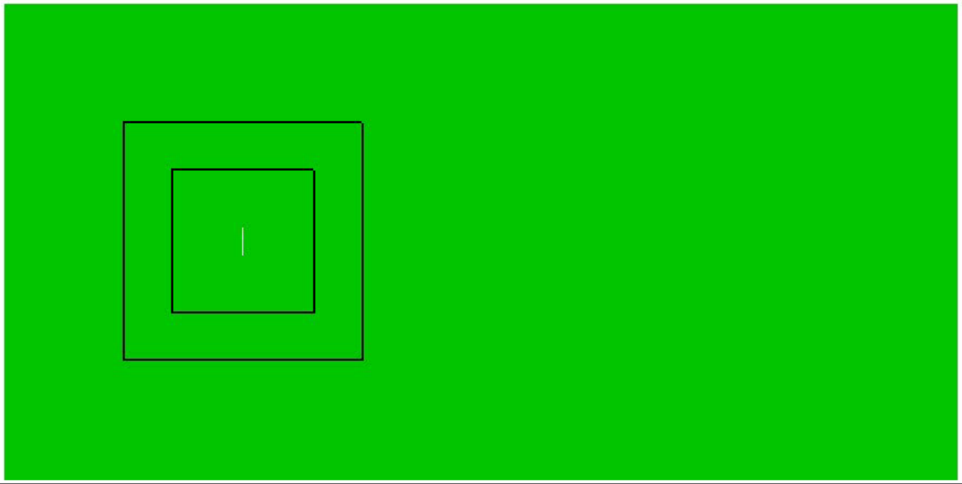


Mach=0.2 flow over vortex generator



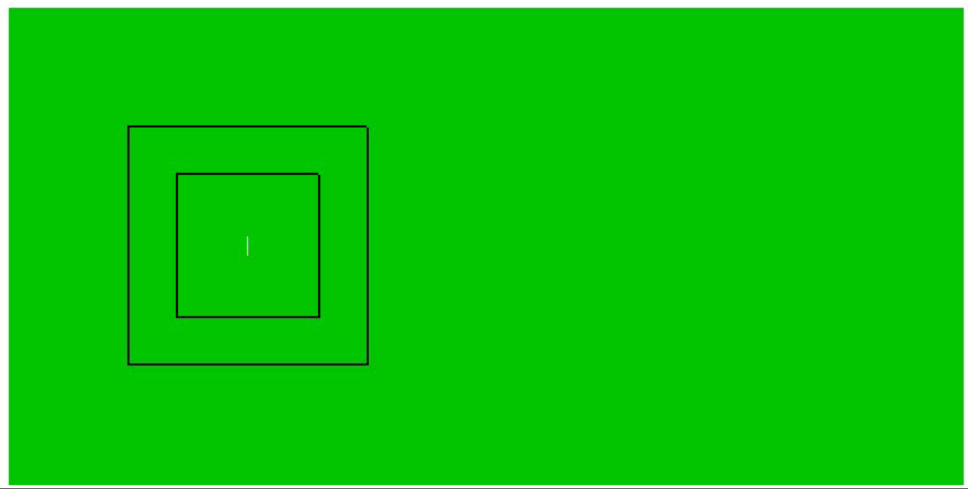


Mach=0.2 flow over vortex generator (vorticity contours)

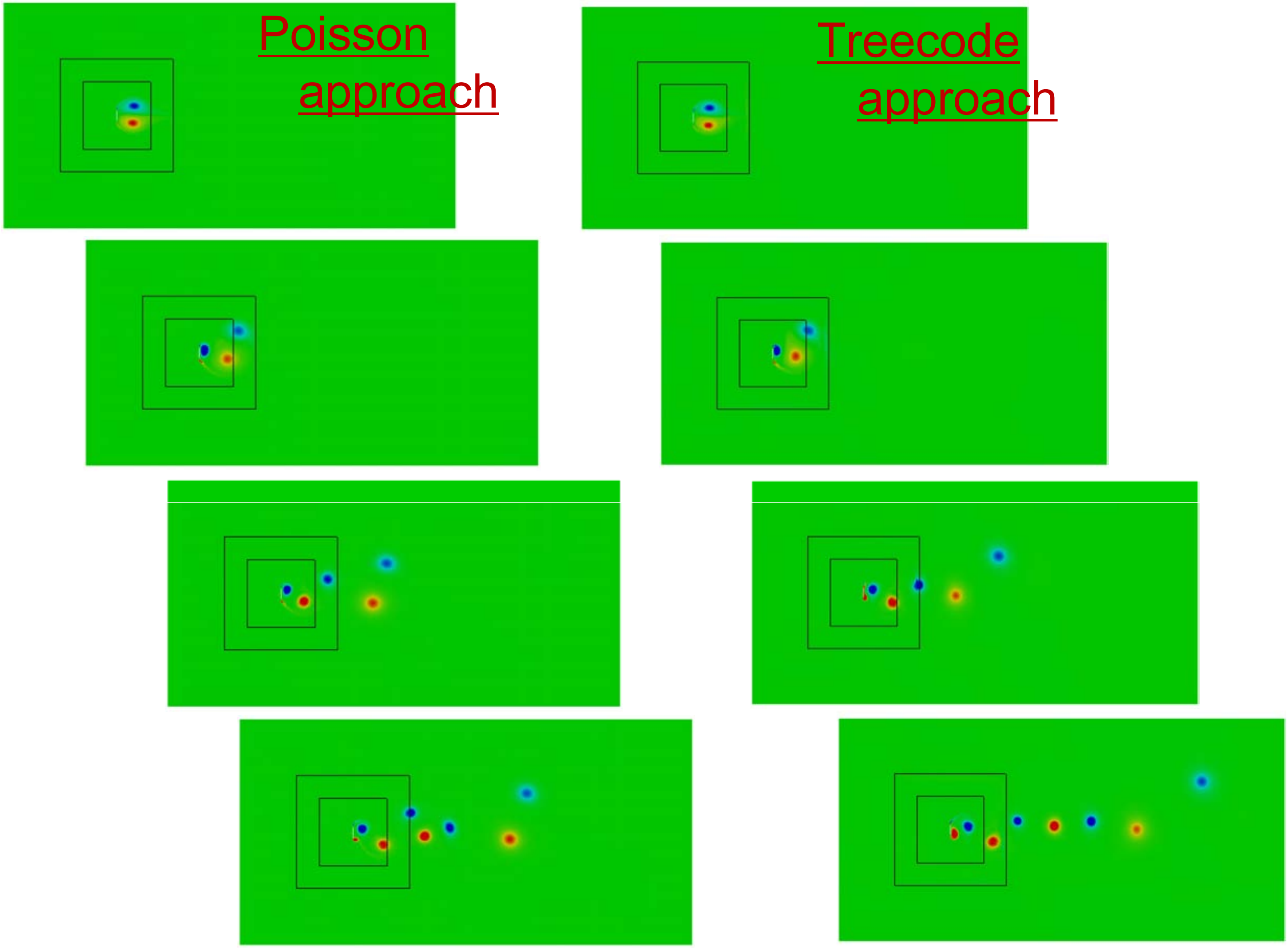


Poisson approach

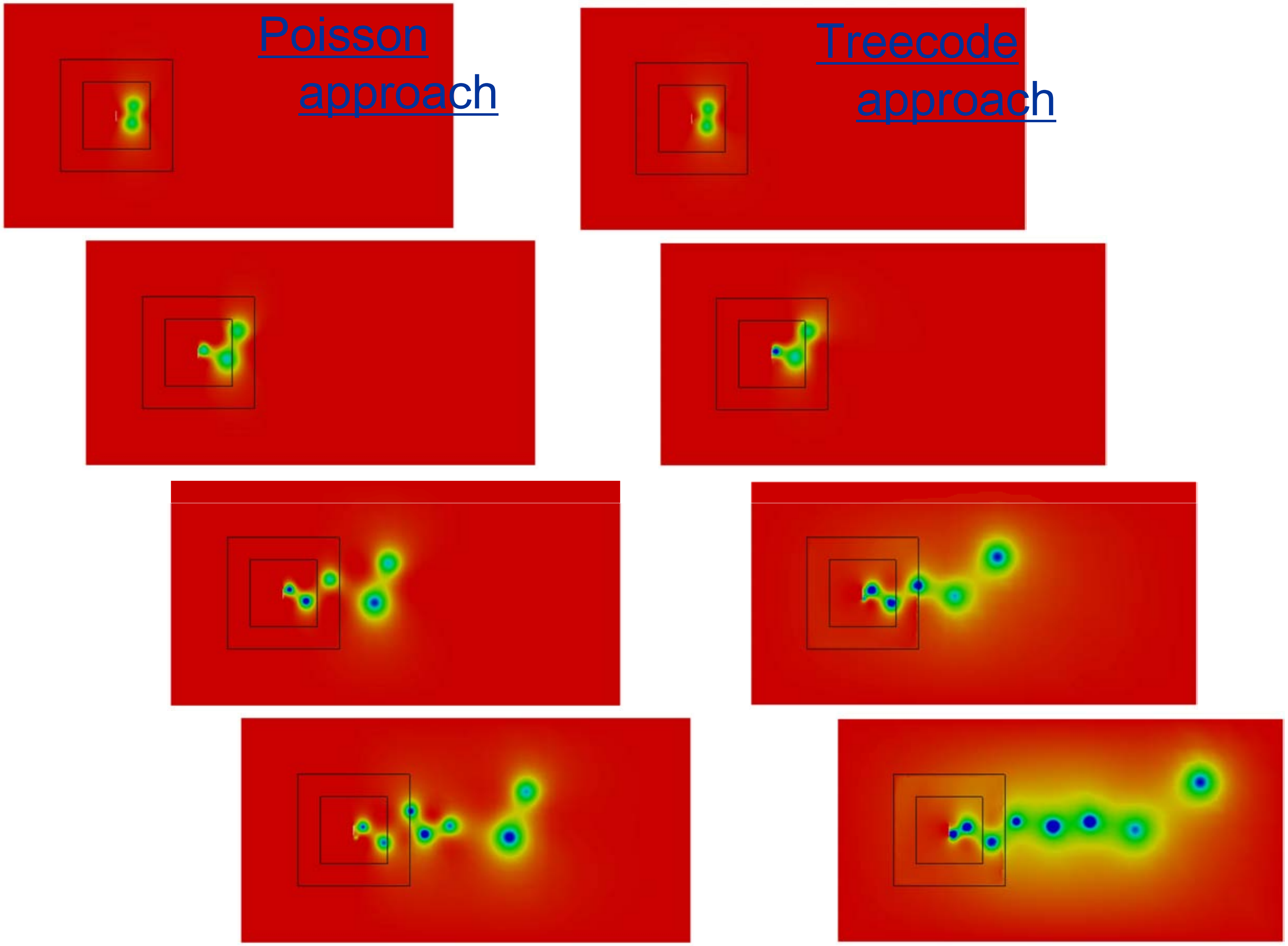
Treecode approach



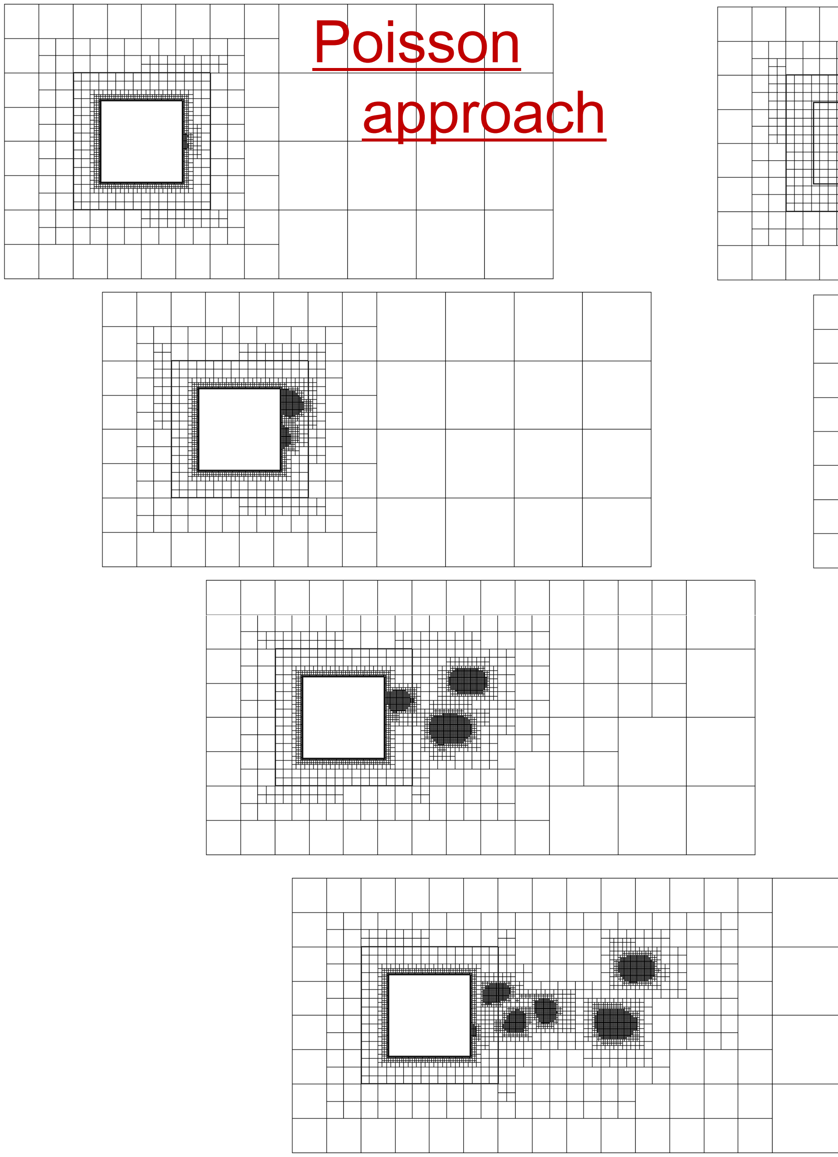
Mach=0.2 flow over vortex generator (vorticity contours)

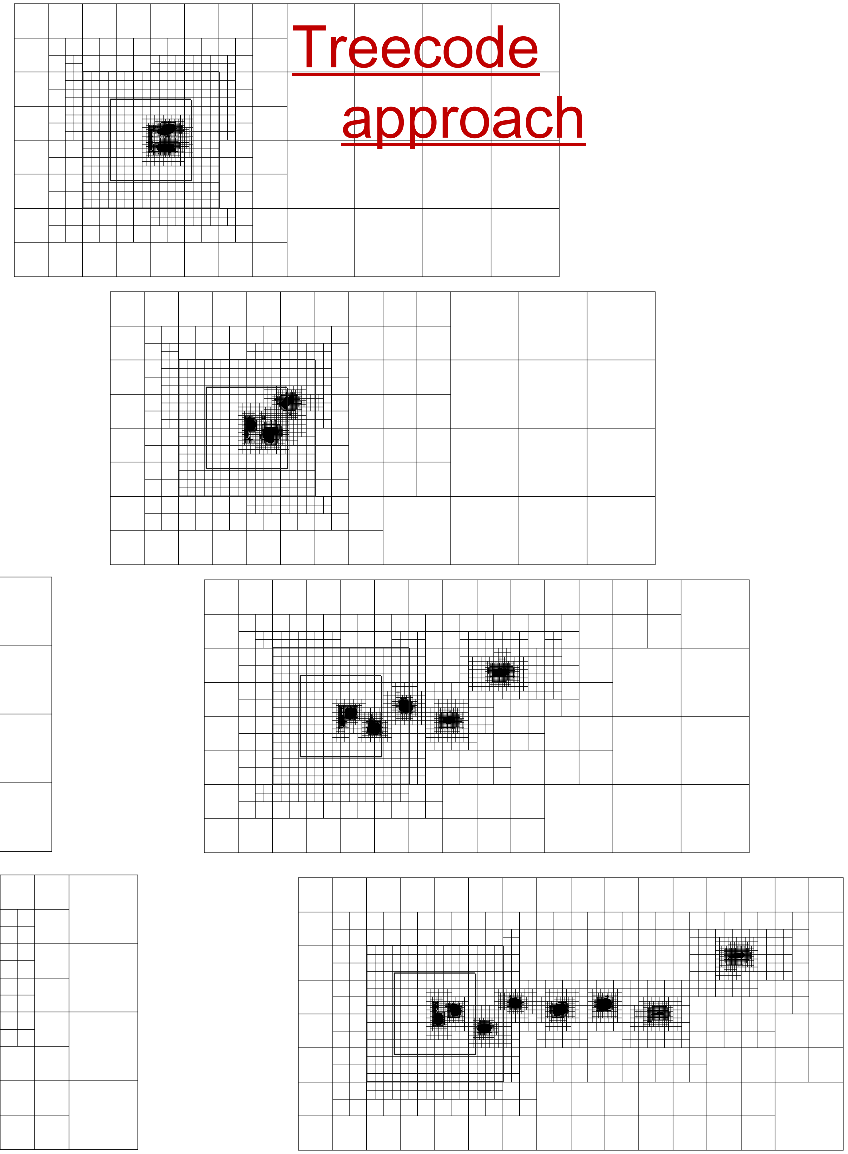


Mach=0.2 flow over vortex generator (vorticity contours)

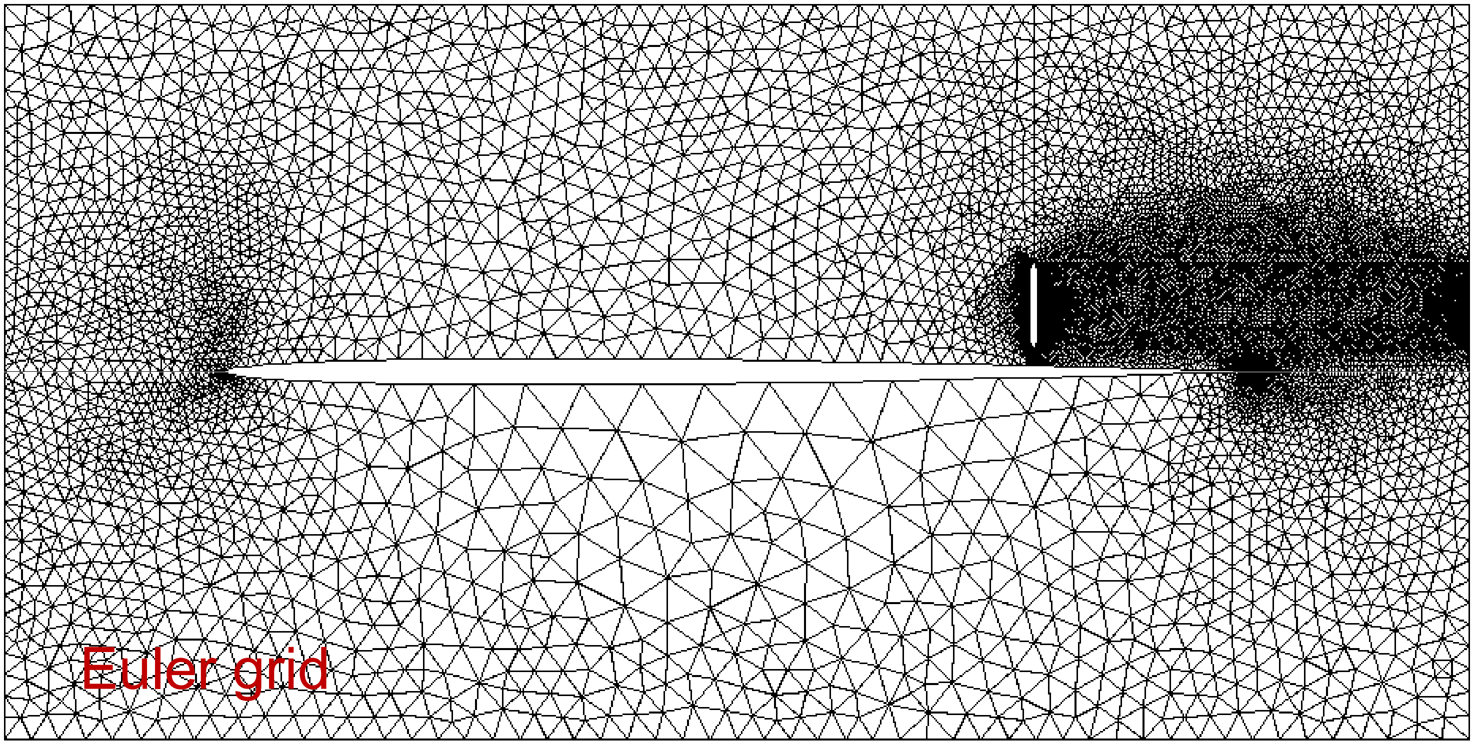


Mach=0.2 flow over vortex generator (grids)

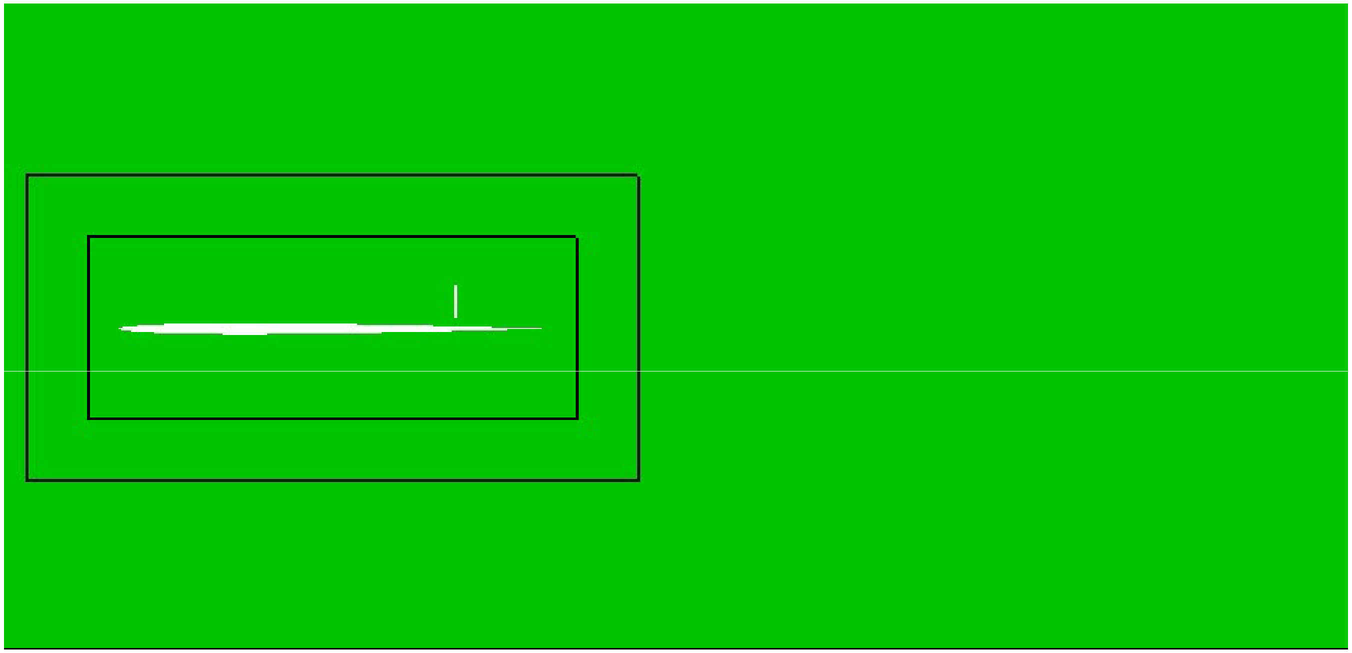




Mach=0.2 flow over NACA 0006 w/ vortex generator

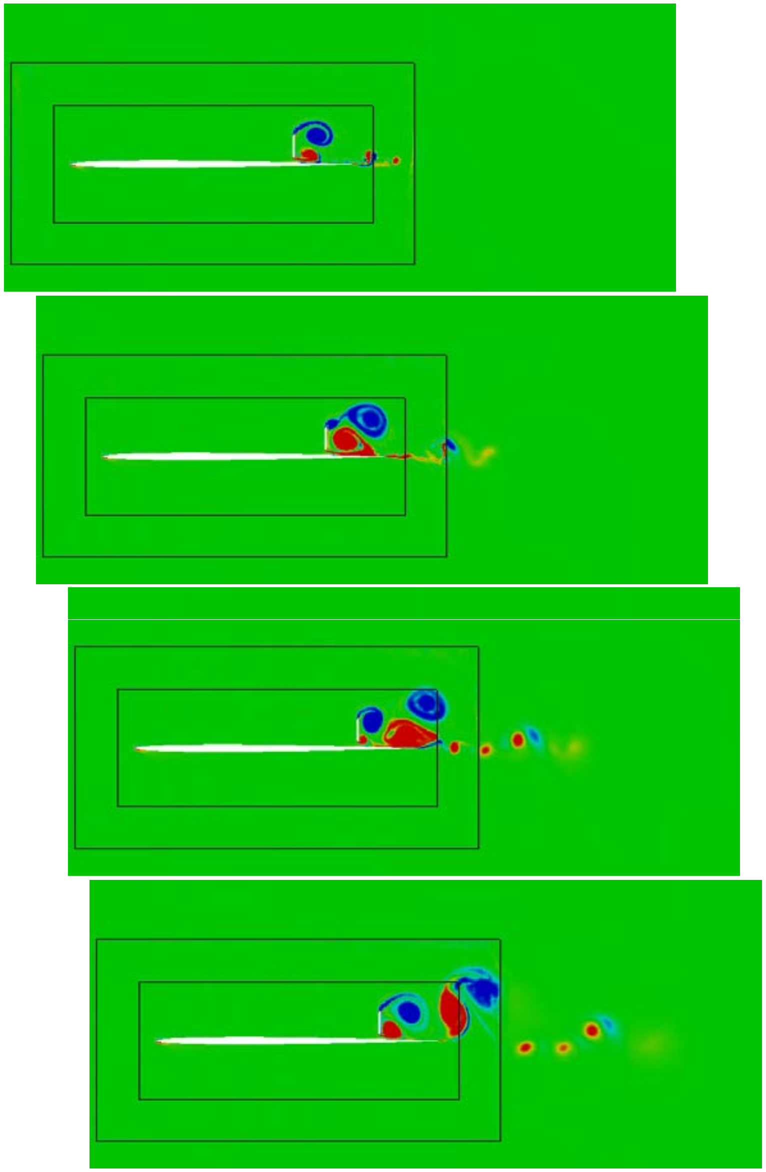


Mach=0.2 flow over NACA 0006 w/ VG (Poisson)



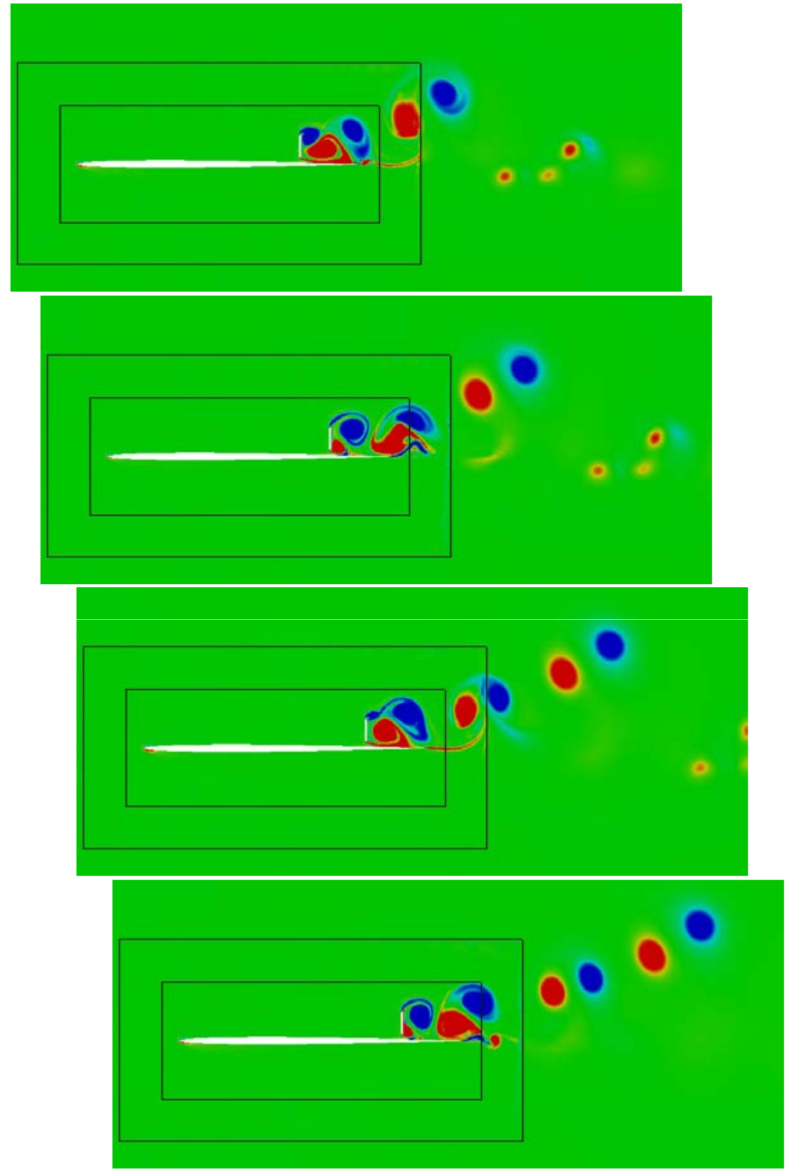
Vorticity contours

Mach=0.2 flow over NACA 0006 w/ VG (Poisson)



Vorticity

contours



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Summary and Conclusions

- Eulerian Vorticity Transport (EVT) solver successfully coupled to compressible Euler solver to accurately simulate vortex-dominated flows
- Overset coupling facilitated by SUGGAR/DiRTlib libraries to minimize necessary modifications to individual solvers
- Vorticity generation in compressible domain, and subsequent convection of vortical structures into EVT domain, was effectively demonstrated for several different vortex-dominated flow problems
- Implemented two different approaches for computing the EVT velocity field and demonstrated effectiveness of both

Ongoing and Future Work

- Coupling of EVT solver with OVERFLOW 2.1 Navier-Stokes solver
- Continued testing and validation of coupling approaches
- Development of Suggar++, including API for communicating EVT grid changes, in progress (Ralph Noack, Penn State ARL)

Questions ???