Multiple-domain, finite-difference solutions for ocean wave-structure interaction

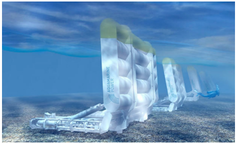
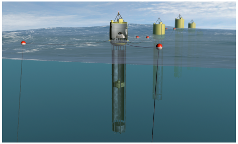
Robert W. Read Harry B. Bingham

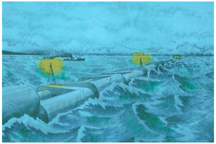
Department of Mechanical Engineering
Technical University of Denmark

10th Symposium on Overset Composite Grids
and Solution Technology
September 23, 2010

Outline

Electricity from ocean waves




- Urgent need to deploy renewable energy technologies
- Ocean wave energy can make significant contribution to mix
- Young industry developing many different design concepts
- Practical computational tools required

Modelling water waves

Flow characteristics

- High Reynolds numbers: $> 10^7$
- Incompressible fluid
- Internal flow is irrotational
- Viscosity significant at walls and during wave breaking

Possible simulation approaches

1. Direct solution of Navier-Stokes equations
2. Inviscid Euler equations
3. Boussinesq-type formulations

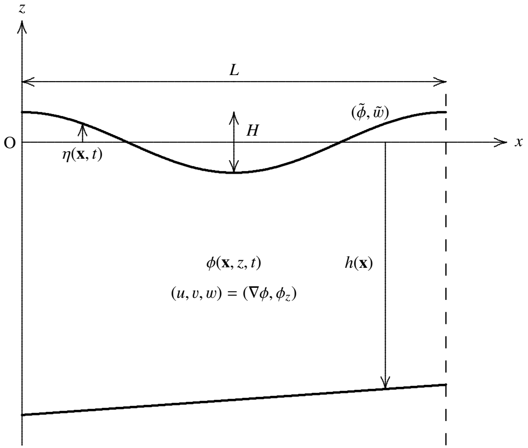
The potential flow approach

Our strategy

- Assume potential flow
- Separate inclusion of friction and wave breaking effects

- Velocity field is gradient of scalar function: velocity potential
- Implies flow is irrotational and thus inviscid and laminar
- No boundary layers or turbulence
- Fluid assumed incompressible

The non-linear wave problem



Governing equations

Bulk fluid

Laplace's equation: $\nabla^2\phi + \partial_{zz}\phi = 0 \quad \text{where} \quad -h \leq z < \eta$

Boundary conditions

1. Free surface
   - Kinematic: $\partial_t\eta = \tilde{w}\,(1 + \nabla\eta\cdot\nabla\eta) - \nabla\tilde{\phi}\cdot\nabla\eta$
   - Dynamic: $\partial_t\tilde{\phi} = -g\eta - {}^1\!/_2\left(\nabla\tilde{\phi}\cdot\nabla\tilde{\phi} - \tilde{w}^2\,(1 + \nabla\eta\cdot\nabla\eta)\right)$
2. Solid surface: $(\boldsymbol{n}, n_z)\cdot(\nabla, \partial_z)\,\phi = 0 \quad \text{where} \quad (\boldsymbol{x}, z) \in \partial\Omega$

The linear formulation

The assumption

The wave elevation is small compared to the wavelength and the free surface velocity is thus vertical.

Reduced equations

1. Laplace's equation: $\nabla^2\phi + \partial_{zz}\phi = 0 \quad \text{where} \quad -h \le z < \eta$
2. Free surface
   - Kinematic: $\partial_t \eta = \tilde{w}$
   - Dynamic: $\partial_t \tilde{\phi} = -g\eta$
3. Solid surface: $(\boldsymbol{n}, n_z) \cdot (\nabla, \partial_z)\,\phi = 0 \quad \text{where} \quad (\boldsymbol{x}, z) \in \partial\Omega$

Modelling objectives

Overall

- Simulate fully non-linear wave-body interaction

Preliminary

- Computational tool describing linear case
- Variable depth and complex body geometries

Immediate

- Extend single-block wave model to multiple curvilinear grids

Today's talk: Linear multi-block solutions for wave-body interaction

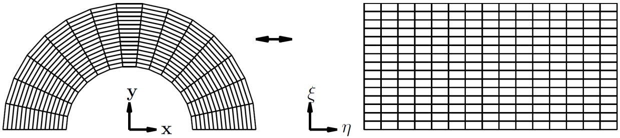
A finite-difference approach

- Require method to identify velocity potential at every grid point
- Represent spatial governing equations in finite-difference form
- Transformations between physical and computational space
- Include interpolation between blocks
- Construct multi-block linear system and solve

Boundary conditions

- Dirichlet condition applied at free-surface points
- Neumann conditions applied at solid-surface ghost points
- Conditions enforced using one of two approaches:
  1. off-centred schemes, single ghost layer at solid surface
  2. centred schemes, multiple ghost layers and extrapolation

Curvilinear transformations



- Mappings: $[\eta(x,y), \zeta(x,y)]$ and $[x(\eta,\zeta), y(\eta,\zeta)]$
- Partial derivatives in two spaces related by:

$$\partial_x = \zeta_x \partial_\zeta + \eta_x \partial_\eta, \quad \partial_y = \zeta_y \partial_\zeta + \eta_y \partial_\eta, \quad \text{etc.}$$

- Express derivatives as operations in computational space:

$$\partial_x = \frac{1}{J}\left(y_\eta \partial_\zeta - y_\zeta \partial_\eta\right) \quad \text{where} \quad J = x_\zeta y_\eta - x_\eta y_\zeta, \quad \text{etc.}$$

A Runge-Kutta approach

- Advance using classical fourth-order Runge-Kutta method

$$\frac{\mathrm{d}}{\mathrm{d}t}\underbrace{\begin{bmatrix} \eta \\ \tilde{\phi} \end{bmatrix}}_{y} = \underbrace{\begin{bmatrix} \partial\tilde{\phi}/\partial z \\ -g\eta \end{bmatrix}}_{f(t,y)}, \quad y_{n+1} = y_n + \frac{\delta t}{6}\left(k_1 + 2k_2 + 2k_3 + k_4\right)$$

- Time step determination using a Courant number of 0.5

$$\text{CFL condition: } u\frac{\Delta t}{\Delta x} \leq C \quad \text{In this case: } \frac{\delta t/T}{\delta x/\lambda} = 0.5$$

Software

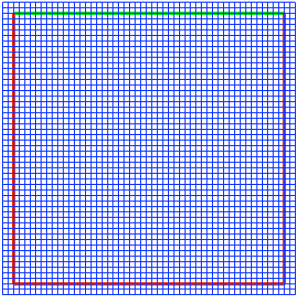
Matlab

- Straightforward implementation, debugging, and testing
- Can maintain total flexibility in spatial order of accuracy

Overture

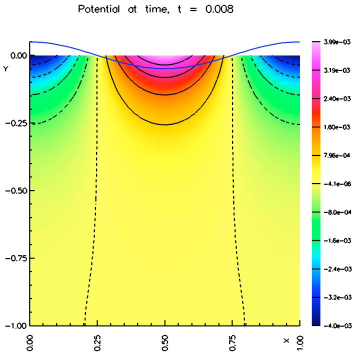
- Code framework to solve pde's on multiple curved blocks
- C++ libraries provide functionality with low-level control
- Powerful grid generation and parallel processing capabilities
- Multiple solvers, but limited to fourth-order spatially

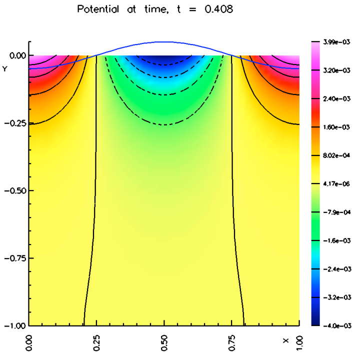
Overture implementation



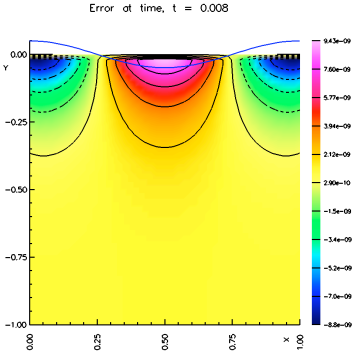
- Unit square, 50 by 50 points
- Sinusoidal surface, $\lambda = 1$ m
- Fourth-order schemes
- Courant number $= 0.5$

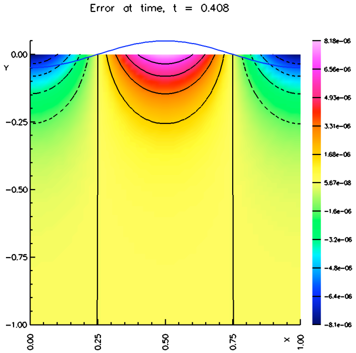
Velocity potential



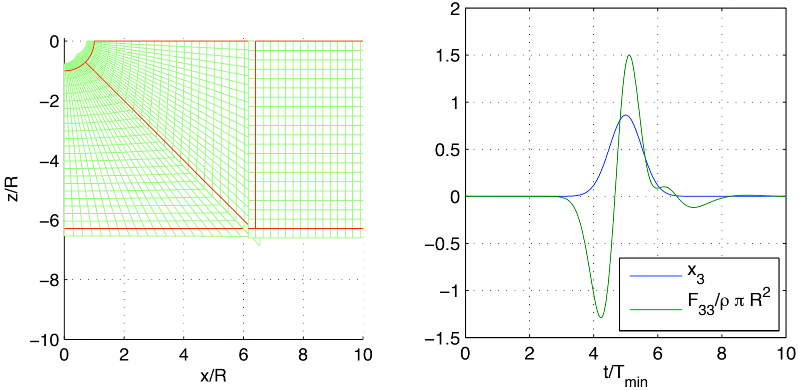


Numerical error



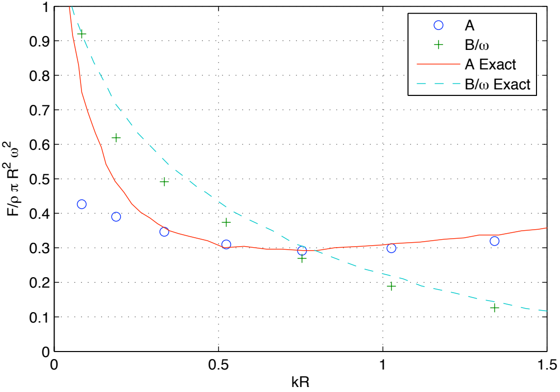


Matlab implementation



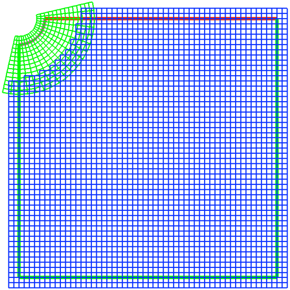
- Vertical flux at cylinder
- Free-surface initially flat
- Sixth-order spatial schemes
- Gaussian displacement profile

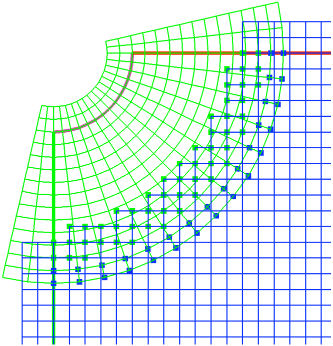
Added mass and damping



- Good agreement between numerical and analytical solution
- Differences in added mass at long wavelengths expected

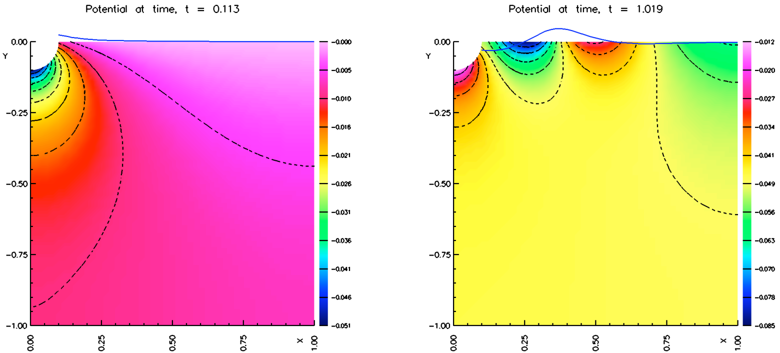
Overture implementation





- Vertical flux at cylinder
- Free-surface initially flat
- Fourth-order schemes
- Manufactured solution testing

The velocity potential



Outlook

Accomplishments

- Implemented linear wave models on curvilinear multiblock geometries
- Evaluated added mass and damping of heaving cylinder

Work in progress - Overture

- Evaluate added mass and damping and compare results
- Investigate using arbitrary-order spatial schemes
- Implement iterative solver
- Further investigate grid generation capabilities and extend to 3-d

Acknowledgements

- This work is supported by the Danish Council for Strategic Research through the Structual Design of Wave Energy Devices project.
- We wish to thank Bill Henshaw for his help and guidance using the Overture package.